7) A conical tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows out of the tank at a rate of 20 ft^3/min, how fast is the depth of the water decreasing when the water is 16 ft deep?

$\frac{dV}{dt} = -20 ft^3/min$

$\frac{dh}{dt} = ?$ when $h = 16$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

$V = \frac{25}{432}\pi h^3$

$\frac{dV}{dt} = \frac{25}{144}\pi h^2 \frac{dh}{dt}$

$-20 = \frac{25}{144}\pi(16)^2 \cdot \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -\frac{9}{20\pi} ft/min}$

$\frac{24}{10} = \frac{h}{r}$

$24r = 10h$

$r = \frac{5}{12}h$

8) A man who is 6 ft tall is walking toward a light at 5 feet per second. The light is 25 feet tall. When the man is 15 feet from the light, at what rate is the tip of his shadow moving? At what rate is the length of his shadow changing?

a) $\boxed{\frac{dx}{dt} + \frac{dy}{dt} = \frac{-125}{19} ft/s}$

b) $\boxed{\frac{dy}{dt} = \frac{-30}{19} ft/s}$

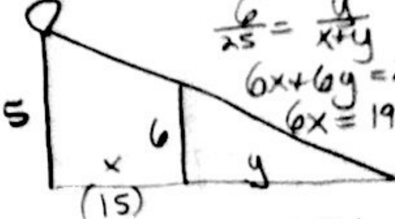
$\frac{dx}{dt} = -5 ft/s$

$6x = 19y$

$6\frac{dx}{dt} = 19\frac{dy}{dt}$

$6(-5) = 19\left(\frac{dy}{dt}\right)$

$\frac{dy}{dt} = -\frac{30}{19} ft/s$



$\frac{6}{25} = \frac{y}{x+y}$

$6x + 6y = 25y$

$6x = 19y$

switch to day 1

9) Skippy and Binky are going to begin a hike at the same location and travel in perpendicular directions. Skippy travels due north at a rate of 5 miles per hour. Binky travels due west at a rate of 8 miles per hour. At what rate is the distance between them changing 3 hours into the hike?

@ $t = 3$

5

$\frac{dy}{dt} = 5$ mph

$\frac{dx}{dt} = -8$mph

when $t = 3$, $\frac{dz}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

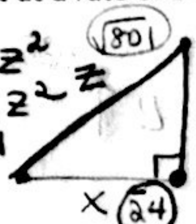
$2(-24)(-8) + 2(15)(5) = 2(\sqrt{801})\frac{dz}{dt}$

$\boxed{\frac{267}{\sqrt{801}} mph} = \frac{dz}{dt}$

$x^2 + y^2 = z^2$

$(-24)^2 + 15^2 = z^2$

$z = \sqrt{801}$



10) A 16-foot ladder leans against a wall. The bottom of the ladder is 5 ft from the wall at time t = 0 and slides away from the wall at a rate of 3 ft/sec. Find the velocity of the top of the ladder at time t = 1.

$z = 16$

$t = 0$, $x = 5$

$t = 1$, $x = 8$

$\frac{dx}{dt} = 3ft/sec$

$\frac{dy}{dt} = ?$

$x^2 + y^2 = z^2 \leftarrow$ constant

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

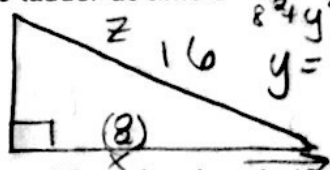
$2(8)(3) + 2(\sqrt{192})\left(\frac{dy}{dt}\right) = 0$

$2\sqrt{192}\frac{dy}{dt} = -48$

$\boxed{\frac{dy}{dt} = \frac{-24}{\sqrt{192}} ft/s}$

$8^2 + y^2 = 16^2$

$y = \sqrt{192}$



11) An airplane is at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance (s) is changing at a rate of 240 mph. What is the speed of the airplane?

when $z = 10$,

$\frac{dz}{dt} = 240$mph

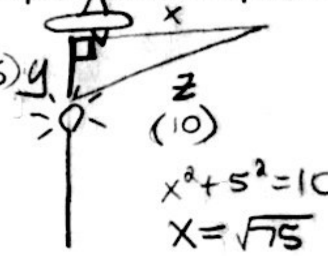
$\frac{dx}{dt} = ?$

$x^2 + y^2 = z^2$ (y is constant)

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

$2(\sqrt{75})\left(\frac{dx}{dt}\right) = 2(10)(240)$

$\boxed{\frac{dx}{dt} = \frac{2400}{\sqrt{75}} mph}$



$x^2 + 5^2 = 10^2$

$x = \sqrt{75}$

switch to day 1

12) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ radians?

2

$z = 10$

$\frac{dx}{dt} = 2ft/s$

$\frac{d\theta}{dt} = ?$ when $\theta = \frac{\pi}{4}$

$\sin\theta = \frac{x}{10}$

$\sin\theta = \frac{1}{10}x$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$

$\cos\frac{\pi}{4}\frac{d\theta}{dt} = \frac{1}{10}(2)$

$\frac{d\theta}{dt} = \frac{1}{5\cos\frac{\pi}{4}}$

$\frac{1}{5 \cdot \frac{\sqrt{2}}{2}}$

$\boxed{= \frac{2}{5\sqrt{2}} rad/s}$