switch w/ #9 Day 2

5) A water tank has the shape of an inverted circular cone with base radius 2 m and a height of 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep?

$\frac{dV}{dt} = 2\ m^3/min$

$\frac{dh}{dt} = ?$ when $h = 3$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$

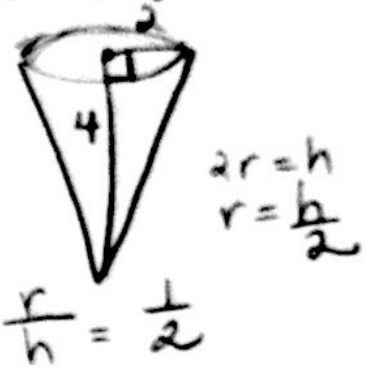
$V = \frac{1}{3}\pi\frac{h^2}{4}$

$V = \frac{h^3}{12}\pi = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$2 = \frac{1}{4}\pi(3)^2\frac{dh}{dt}$

$\frac{2}{9\pi}\ m/min$



6) A ladder 26 feet long is resting against the side of a house. The foot of the ladder is pulled away at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 10 ft from the wall?

(6) $\frac{dx}{dt} = 4\ ft/sec$

$z = 26$

$\frac{dy}{dt} = ?$ when $x = 10$

$x^2 + y^2 = 26^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(10)(4) + 2(24)\left(\frac{dy}{dt}\right) = 0$

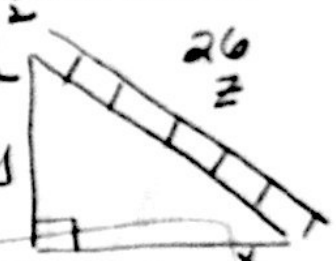
$48\frac{dy}{dt} = -80$

$\frac{dy}{dt} = -\frac{5}{3}\ ft/sec$

$x^2 + y^2 = 26^2$

$10^2 + y^2 = 26^2$

$y = 24$



switch w/ #12 Day 2

7) A hot air balloon is rising at a rate of 9 yards per second. From a point 15 yards away on the ground, an observer measures the height of the balloon. What is the rate of change of the angle from the ground to the balloon when the balloon is 100 yards high?

$\frac{dy}{dt} = 9\ yd/s$

$x = 15$

$\frac{d\theta}{dt} = ?$

$y = 100$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{y}{15}$

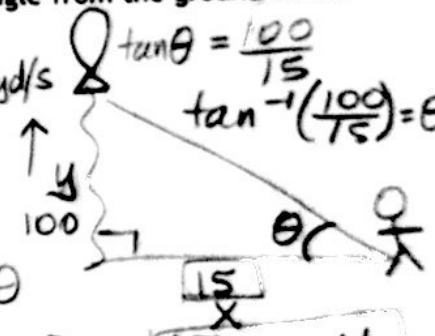
$\tan\theta = \frac{1}{15}y$

$\cos^2\theta \cdot \sec^2\theta\frac{d\theta}{dt} = \frac{1}{15}\frac{dy}{dt}\cdot\cos^2\theta$

$\frac{d\theta}{dt} = \frac{1}{15}(9)\cdot\cos^2\left(\tan^{-1}\left(\frac{100}{15}\right)\right) = \frac{27}{2045}\ rad/s$

$\tan\theta = \frac{100}{15}$

$\tan^{-1}\left(\frac{100}{15}\right) = \theta$



8) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

(8) $\frac{dz}{dt} = -1\ m/s$

$y = 1$

when $x = -8$,

$\frac{dx}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 1 = 2z\frac{dz}{dt}$

$\frac{2(-8)\left(\frac{dx}{dt}\right)}{16} = \frac{2(\sqrt{65})(1)}{16}$

$\frac{dx}{dt} = \frac{\sqrt{65}}{8}\ m/s$

$x^2 + y^2 = z^2$

$(-8)^2 + 1^2 = z^2$

$z = \sqrt{65}$