Sammit

AP Calculus AB

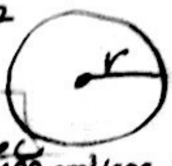
2.6 Related Rates Day 2

Name: Key

1) Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft?

$\frac{dr}{dt} = 2 \text{ ft/sec}$ $\qquad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\qquad A = \pi r^2$

$\frac{dA}{dt} = ?$ when $r = 60$ $\qquad \frac{dA}{dt} = 2\pi(60)(2) = \boxed{240\pi \text{ ft}^2/\text{sec}}$



2) Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm³/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 100 \text{ cm}^3/\text{sec}$ $\qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = ?$ when $r = 25$ $\qquad 100 = 4\pi(25)^2 \cdot \frac{dr}{dt}$ $\qquad \boxed{\frac{1}{25\pi} \text{ cm/sec}}$

$d = 50$, $r = 25$

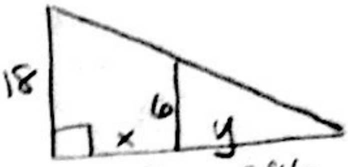
3) A man 6 feet tall is walking at a rate of 3 ft/s toward a street light 18 ft high. At what rate is the tip of the shadow moving? At what rate is the shadow length changing?

$\frac{6}{18} = \frac{y}{x+y}$

a) $\frac{dx}{dt} + \frac{dy}{dt} = \boxed{-\frac{9}{2} \text{ ft/s}}$ $\qquad \frac{dx}{dt} = -3 \text{ ft/s}$ $\qquad 6\frac{dx}{dt} = 12\frac{dy}{dt}$ $\qquad 6x + 6y = 18y$

b) $\frac{dy}{dt} = \boxed{-\frac{3}{2} \text{ ft/s}}$ $\qquad 6(-3) = 12\left(\frac{dy}{dt}\right)$ $\qquad 6x = 12y$

$\frac{dy}{dt} = -\frac{3}{2} \text{ ft/s}$



-3 ft/s

4) Gas is escaping from a spherical balloon at the rate of 2 ft³/min. How fast is the surface area changing when the radius is 12 ft?

$V = \frac{4}{3}\pi r^3$ $\qquad A = 4\pi r^2$ $\qquad V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = -2 \text{ ft}^3/\text{min}$ $\qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\qquad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = ?$ $\qquad -2 = 4\pi(12)^2 \frac{dr}{dt}$ $\qquad \frac{dA}{dt} = 8\pi(12)\left(\frac{-1}{288\pi}\right)$

when $r = 12$ $\qquad \frac{dr}{dt} = \frac{-1}{288\pi} \text{ ft/min}$ $\qquad = \boxed{-\frac{1}{3} \text{ ft}^2/\text{min}}$

5) Sand falls from a conveyer belt at the rate of 10 m³/min onto the top of a conical pile. The height of the pile is always 3/8 of the base of the diameter. How fast are the height and radius changing when the pile is 4 m high?

$V = \frac{1}{3}\pi r^2 h$ $\qquad 10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \frac{dr}{dt}$ $\qquad h = \frac{3}{8}d$

$\frac{dV}{dt} = 10 \text{ m}^3/\text{min}$ $\qquad V = \frac{1}{3}\pi r^2\left(\frac{3}{4}r\right)$ $\qquad \boxed{\frac{15\pi}{32} \text{ m/min}} = \frac{dr}{dt}$ $\qquad h = \frac{3}{8}(2r)$

$\frac{dh}{dt} = ?$ $\quad \frac{dr}{dt} = ?$ $\qquad V = \frac{1}{4}\pi r^3$ $\qquad \frac{dh}{dt} = \frac{3}{4} \cdot \frac{15\pi}{32}$ $\qquad h = \frac{3}{4}r$

$\frac{dV}{dt} = \frac{3}{4}\pi r^2 \frac{dr}{dt}$ $\qquad h = \frac{3}{4}r$

$h = 4$ $\qquad \frac{dh}{dt} = \frac{3}{4}\frac{dr}{dt}$ $\qquad \boxed{\frac{dh}{dt} = \frac{45}{128\pi} \text{ m/min}}$ $\qquad h = 4, r =$

6) The radius of a right circular cylinder is increasing at a rate of 2 in/min and the height is decreasing at a rate of 3 in/min. At what rate is the volume changing when the radius is 8 in and the height is 12 in? Is the volume increasing or decreasing?

$V = \pi r^2 \cdot h$ $\qquad 4 = \frac{3}{4}r$

$\frac{dr}{dt} = 2 \text{ in/min}$ $\qquad \frac{dV}{dt} = \pi\left(r^2 \cdot 1\frac{dh}{dt} + h \cdot 2r\frac{dr}{dt}\right)$ $\qquad 16 = 3r$

$\frac{dh}{dt} = -3 \text{ in/min}$ $\qquad \frac{dV}{dt} = \pi\left(8^2 \cdot -3 + 12 \cdot 2(8)(2)\right)$ $\qquad r = \frac{16}{3}$

$\frac{dV}{dt} = ?$ when $r = 8, h = 12$ $\qquad \frac{dV}{dt} = \boxed{192\pi \text{ in}^3/\text{min} \text{ increasing}}$