AP Calculus AB
2.6 Related Rates Day 1

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Strategies for Solving Related Rates Problems:

- Read the problem carefully.
- Draw a diagram if possible.
- Introduce notation. Assign symbols to all quantities that are functions of time.
- Express the given information and the required rate in terms of derivatives.
- Write an equation that relates the various quantities of the problem. If necessary, use geometry to eliminate one of the variables by substitution.
- Use the Chain Rule to differentiate both sides of the equation with respect to t.
- Substitute the given information in the resulting equation and solve for the unknown rate.

1) A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a constant rate of 1 ft/sec. When the radius is 4 ft, at what rate is the total area of the disturbed water changing?

$\frac{dr}{dt} = 1$ ft/sec
$r = 4$
$\frac{dA}{dt} = ?$

$A = \pi r^2$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
$\frac{dA}{dt} = 2\pi(4)(1) =$ 8π ft^2/sec

2) Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius when the radius is 2 inches.

1) $\frac{dV}{dt} = 4.5$ in^3/min
$\frac{dr}{dt} = ?$
$r = 2$

$V = \frac{4}{3}\pi r^3$
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$4.5 = 4\pi(2)^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{9}{32\pi}$ in/min

3) Water runs into a conical tank at the rate of 2 ft^3/min. The tank stands with the point down and has a height of 10 feet and a base diameter of 10 feet. How fast is the water level rising when the water is 6 feet deep?

3) $\frac{dV}{dt} = 2$ ft^3/min
$h = 10$
$d = 10, r = 5$
when $h = 6$, $\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$
$V = \frac{1}{3}\pi \cdot \frac{1}{4}h^3$
$V = \frac{1}{12}\pi h^3$
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{r}{h} = \frac{5}{10}$
$10r = 5h$
$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$2 = \frac{1}{4}\pi(6)\cdot\frac{dh}{dt}$
$\frac{2}{9\pi}$ ft/min

4) A woman 6 feet tall walks at a rate of 3 feet/sec away from a light that is 45 feet tall. When she is 25 feet away from the light, at what rate is the tip of her shadow moving? At what rate is the length of her shadow changing?

4) $x = 25$
$\frac{dx}{dt} = 3$ ft/sec

a) $3 + \frac{6}{13} =$ $\frac{45}{13}$ ft/sec

b) $\frac{6}{13}$ ft/sec

$\frac{6}{45} = \frac{y}{x+y}$
$6x + 6y = 45y$
$6x = 39y$
$6\frac{dx}{dt} = 39\frac{dy}{dt}$
$6(3) = 39\frac{dy}{dt}$
$\frac{6}{13} = \frac{dy}{dt}$