Sammit-key
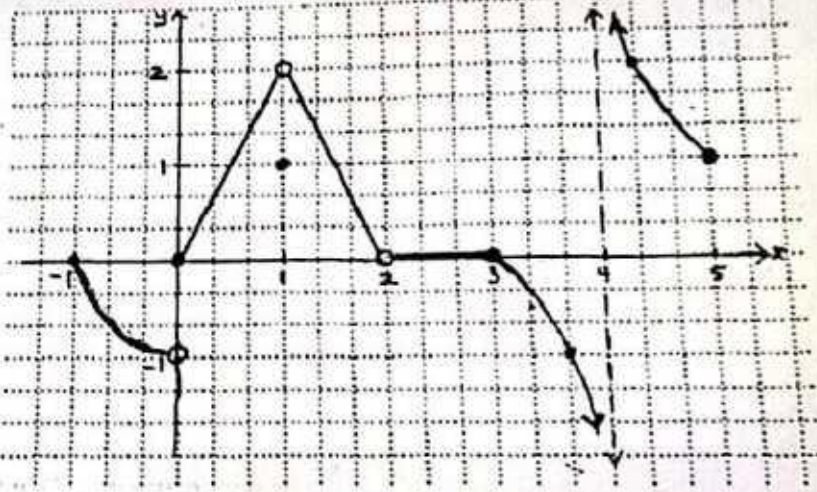
## The Meaning of Continuity:

Consider the function $f$ defined below:

$$f(x) = \begin{cases} x^2-1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x=1 \\ -2x+4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \\ \dfrac{1}{2(x-4)}+0.5, & 3 < x \le 5, \ x \ne 4 \end{cases}$$



1. Over the interval [-1, 5], where is the function $f$ discontinuous?

   at $x = 0, 1, 2, 4$

2. Complete the table for each value of x = c given.

| X = c | $f(c)$ | $\lim\limits_{x \to c^-} f(x)$ | $\lim\limits_{x \to c^+} f(x)$ | $\lim\limits_{x \to c} f(x)$ | Does $f(c)=\lim\limits_{x \to c} f(x)$ | Is $f(x)$ continuous at x = c? |
|---|---|---|---|---|---|---|
| 0 | O | −1 | O | DNE | No | No |
| 1 | 1 | 2 | 2 | 2 | No | No |
| 2 | DNE | O | O | O | No | No |
| 3 | O | O | O | O | Yes | Yes |
| 4 | DNE | $-\infty$ | $\infty$ | DNE | DNE N/A | No |

Using the table above, determine the three conditions that must exist in order for a function to be continuous:

a) The $\lim\limits_{x \to c} f(x)$ __exists__

b) $f(c)$ __is defined__

c) $\lim\limits_{x \to c} f(x)$ __equals__ $f(c)$

**EXAMPLE:** Let $f$ be the function defined by $f(x) = \begin{cases} x+1, & x<2 \\ k(x-5)^2, & x \ge 2 \end{cases}$

where $k$ stands for a constant.

$x+1 ; x < 2$
$(x-5)^2 , x \ge 2$

1. Graph this function for $k=1$.

2. Use the definition of continuity to explain formally what must be true in order for $f$ to be continuous at $x = 2$.

   The limit has to exist
   $\lim\limits_{x \to 2} = f(2)$



3. Find the value of $k$ that makes $f$ continuous at $x = 2$. Then sketch $f$ for this value of $k$.

$f(2)=9$     $x+1 = k(x-5)^2$
$\lim\limits_{x \to 2} = DNE$   $2+1 = k(2-5)^2$

$3 = k(-3)^2$
$3 = 9k$
$\frac{3}{9} = \frac{9k}{9}$

$\boxed{k = \frac{1}{3}}$