

Higher Order Derivatives:

$$1) \frac{d^2 y}{dx^2} \quad x^2 y^2 = 2x - 3$$

$$x^2 \cdot 2y \frac{dy}{dx} + y^2 \cdot 2x = 2$$

$$\frac{x^2 2y \frac{dy}{dx}}{x^2 2y} = \frac{2 - 2xy^2}{x^2 2y}$$

$$* \frac{dy}{dx} = \frac{-xy^2 + 1}{x^2 y} = -\frac{y}{x} + x^{-2} y^{-1}$$

$$\frac{x \left(-1 \frac{dy}{dx}\right) - y(1)}{x^2} + x^{-2} \cdot -1 y^{-2} \frac{dy}{dx} + y^{-1} \cdot -2x^{-3}$$

$$\frac{d^2 y}{dx^2} = \frac{-x \frac{dy}{dx} + y}{x^2} + -x^{-2} y^{-2} \frac{dy}{dx} - 2x^{-3} y^{-1}$$

$$2) y^2 = x^3. \text{ Find } \frac{d^2 y}{dx^2}$$

$$2y \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\frac{d^2 y}{dx^2} = \frac{(2y)(6x) - (3x^2)(2 \frac{dy}{dx})}{(2y)^2}$$

$$= \frac{12xy - 6x^2 \left(\frac{3x^2}{2y}\right)}{4y^2}$$

$$= \frac{12xy - \frac{18x^4}{2y}}{4y^2} = \frac{\frac{24xy^2}{2y} - \frac{18x^4}{2y}}{4y^2}$$

$$\begin{aligned} & \frac{3(4xy^2 - 3x^4)}{2y} \cdot \frac{1}{4y^2} \\ & = \frac{3(4xy^2 - 3x^4)}{4y^3} \end{aligned}$$