# 9.1 Sequences

1. • sequence - list of numbers written in an explicit order

$$a_n = a_1, a_2, a_3, a_4 \ldots a_n$$

• arithmetic sequence - common difference between terms

recursive: $a_n = a_{n-1} + d$

explicit: $a_n = a_1 + d(n-1)$

• geometric sequence - common ratio between terms

recursive: $a_n = a_{n-1} \cdot r$

explicit: $a_n = a_1 \cdot r^{n-1}$

* Sequence converges if there comes a point when the terms of the sequence remain the same

* Sequence diverges if terms of sequence continue to change

2. To Determine: Find limit as $n \to \infty$

• 4. 1) $a_n = \frac{2n-1}{n}$

$\lim_{n \to \infty} \frac{2n-1}{n}$

$\lim_{n \to \infty} \frac{2\cancel{n}}{\cancel{n}} - \frac{1}{n}$

$= 2 - 0 = \boxed{2}$

∴ converges

OR L'H: $\frac{2}{1} = 2$

→ PROPERTIES!

Converge or Diverge? ⑤ YT

$\hookrightarrow$ Find limit

2) $a_n = n(n+1)$

$\lim_{n\to\infty} n(n+1)$

$\therefore$ diverges

3) $a_n = \frac{3+5n^2}{n+n^2}$

$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2}$

L'H: $\frac{10n}{2n} = 5$

$\therefore$ converges

4) $a_n = \cos\left(\frac{n}{2}\right)$

$\lim_{n\to\infty} \cos\left(\frac{n}{2}\right) = \text{DNE}$

$\therefore$ diverges

5) $a_n = \frac{e^n + e^{-n}}{e^{2n} - 1}$

$\lim_{n\to\infty} \frac{e^n + e^{-n}}{e^{2n} - 1}$

L'H: $\frac{e^n - e^{-n}}{2e^{2n}} = 0$

$\therefore$ converges

6) $a_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore$ converges

7) $a_n = \left(1+\frac{1}{n}\right)^n$

$y = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \quad 1^\infty$

$\ln y = \lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right)$

$\ln y = \lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$

L'H: $\ln y = \lim_{n\to\infty} \frac{\frac{1}{1+\frac{1}{n}} \cancel{\left(\frac{-1}{n^2}\right)}}{\cancel{\left(\frac{-1}{n^2}\right)}} = 1$

$\ln y = 1$

$y = e \quad \therefore$ converges

3 Properties of Limits of Sequences:

Let $\lim_{n\to\infty} a_n = L$ $\lim_{n\to\infty} b_n = K$

1) $\lim_{n\to\infty} (a_n \pm b_n) = L \pm K$

2) $\lim_{n\to\infty} c a_n = cL$, c is any real #

3) $\lim_{n\to\infty} (a_n b_n) = LK$

4) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L}{K}$ $b_n \neq 0$, $K \neq 0$

Abs Value Example:

$a_n \quad \frac{(-1)^n}{n}$ $\lim_{n\to\infty} \frac{(-1)^n}{n}$

$\lim_{n\to\infty} \left| \frac{(-1)^n}{n} \right| = \lim_{n\to\infty} \frac{1}{n} = 0$

then

$\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$

• 8) $a_n = \sin n$

$\lim_{n\to\infty} \sin n = DNE$

$\therefore$ diverges

9) $a_n = \dfrac{1-5n^4}{n^4+8n^3}$

$\lim_{n\to\infty} \dfrac{1-5n^4}{n^4+8n^3}$

L'H: $\dfrac{-20n^3}{4n^3}$

L'H: $\dfrac{-60n^2}{12n^2}$

L'H: $\dfrac{-120n}{24n}$

L'H: $-\dfrac{120}{24} = -5$

$\therefore$ converges

10) $a_n = \dfrac{n^2-2n+1}{n-1}$

$\lim_{n\to\infty} \dfrac{(n-1)\cancel{(n-1)}}{\cancel{(n-1)}} = DNE$

$\therefore$ diverges

• 11) $a_n = \left(\dfrac{n+1}{2n}\right)\left(1-\dfrac{1}{n}\right)$

$\lim_{n\to\infty} \left(\dfrac{1}{2}\right)\cdot(1)$

$= \dfrac{1}{2}$

$\therefore$ converges

12) $a_n = \dfrac{n^2}{2^n-1}$

$\lim_{n\to\infty} \dfrac{n^2}{2^n-1}$ huge $= 0$

$\therefore$ converges

• 13) $a_n = 1 + \dfrac{(-1)^n}{n}$

$\lim_{n\to\infty} \left(1+\dfrac{(-1)^n}{n}\right)$

$= 1+0 = 1$

$\therefore$ converges

⑥ Absolute Value Theorem for Sequences:
If the abs value of the terms of a sequence converge to 0, then the sequence converges to 0

If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$

C or D?

1) $a_n = \dfrac{(-1)^{n-1} \cdot n}{n^2+1}$

$\lim_{n\to\infty} \dfrac{(-1)^{n-1} \cdot n}{n^2+1} = 0$

big

2) $a_n = 1 + (-1)^n$

diverges

3) $a_n = (-1)^n \cdot \dfrac{n+1}{n}$

diverges

4) $a_n = \dfrac{1+(-1)^n}{n^2}$ big

Converges

5) $a_n = \dfrac{(-3)^n}{n!}$ big

Converges

6) $a_n = \dfrac{(n-2)!}{n!}$

$\dfrac{\cancel{(n-2)}\cancel{(n-3)}\cancel{(n-4)}\ldots}{n(n-1)\cancel{(n-2)}\cancel{(n-3)}\cancel{(n-4)}}$

$\dfrac{1}{n(n-1)} = 0$

converges

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# 9.2 Geometric Series

- infinite sequence – list of terms that continue to infinity
- infinite series – sum of sequence

$$\sum_{K=1}^{\infty} a_K$$

  - partial sum $S_n = \sum_{K=1}^{n} a_K$
  - If $S_n$ has a $\lim_{n \to \infty}$, series converges. Otherwise, diverges

- geometric series – each term is found by multiplying preceding term by $r$

$$\sum_{n=0}^{\infty} ar^n \quad \underline{\text{OR}} \quad \sum_{n=1}^{\infty} ar^{n-1}$$

* If $|r| > 1$, diverges

* If $|r| < 1$, converges to $\frac{a}{1-r}$

1) $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} \ldots$

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$r = \frac{1}{10}$

$a = \frac{3}{10}$

$$\frac{\frac{3}{10}}{1 - (\frac{1}{10})} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{3}{9} = \boxed{\frac{1}{3}}$$

2) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8} \ldots$

$a = 1$

$r = -\frac{1}{2}$

$$\frac{1}{1 - (-\frac{1}{2})} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$$

10 Converge or Diverge?

$\hookrightarrow$ Find sum

3) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{4}{3}\right)^n$

$r=\frac{4}{3}$

$\therefore$ diverges

4) $\displaystyle\sum_{n=0}^{\infty}2\left(\frac{-2}{3}\right)^n$

$r=\frac{-2}{3}$

$\frac{2}{1-\left(-\frac{2}{3}\right)}=\frac{2}{\frac{5}{3}}=\boxed{\frac{6}{5}}$

5) $\displaystyle\sum_{n=0}^{\infty}\frac{3}{2^{n-1}}$

$\frac{3}{2^n\cdot 2^{-1}}=3\cdot 2\left(\frac{1}{2}\right)^n$

$=6\left(\frac{1}{2}\right)^n$

$\frac{6}{1-\left(\frac{1}{2}\right)}=\frac{6}{\frac{1}{2}}=\boxed{12}$

6) $\displaystyle\sum_{n=0}^{\infty}\frac{4}{2^n}$

$\frac{4}{2^n}=4\left(\frac{1}{2}\right)^n$

$\frac{4}{1-\left(\frac{1}{2}\right)}=\boxed{8}$

7) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n}{100}$

$\frac{2^n}{100}=\frac{1}{100}\left(2^n\right)$

$r=2$

$\therefore$ diverges

8) $\displaystyle\sum_{n=0}^{\infty}\frac{5^{n+2}}{4^{n-1}}$

$\frac{5^n\cdot 5^2}{4^n\cdot 4^{-1}}=100\left(\frac{5}{4}\right)^n$

$\therefore$ diverges

⑧ Properties of Infinite Series:

$$\sum a_n = A \qquad \sum b_n = B$$

1) $\sum c \cdot a_n = c \cdot \sum a_n = CA$

2) $\sum (a_n \pm b_n) = \sum a_n \pm \sum b_n$

$= A \pm B$