Ex 2) Find f' for $f(x) = \frac{4}{\sqrt{x}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{4}{\sqrt{x+h}}\cdot \sqrt{x}\sqrt{x+h} - \frac{4}{\sqrt{x}}\cdot\sqrt{x}\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{(4\sqrt{x} - 4\sqrt{x+h})}{(h\sqrt{x}\sqrt{x+h})} \cdot \frac{(4\sqrt{x} + 4\sqrt{x+h})}{(4\sqrt{x} + 4\sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{\overset{\cancel{16x}-\cancel{16x}-16h}{16x - 16(x+h)}}{(h\sqrt{x}\sqrt{x+h})(4\sqrt{x} + 4\sqrt{x+h})} = \lim_{h \to 0} \frac{-16\cancel{h}}{(\cancel{h}\sqrt{x}\sqrt{x+h})(4\sqrt{x} + 4\sqrt{x+h})}$$

$$= \frac{-16}{\sqrt{x}\sqrt{x}(4\sqrt{x} + 4\sqrt{x})} = \frac{-16}{x(8\sqrt{x})} = \boxed{\frac{-2}{x\sqrt{x}}}$$

Ex 3) Find the slope and the eqn of the tangent line for $f(x) = x^2+1$ at $(-1, 2)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \underset{\text{slope}}{\boxed{2x}}$$

slope at $(-1, 2) = 2(-1) = -2$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 2 = -2(x+1)} \quad \text{or} \quad \begin{array}{l} y - 2 = -2x - 2 \\ y = -2x \end{array}$$