8.2 Integration by Parts

"product rule" for integration

$$\int u\,dv = uv - \int v\,du$$

Ultra Violet
Voo-Doo!

Proof:

$$\frac{d}{\cancel{dx}} uv = u\frac{dv}{\cancel{dx}} + v\frac{du}{\cancel{dx}}$$

$$d\,uv = u\,dv + v\,du$$

$$d\,uv - v\,du = u\,dv$$

$$\int u\,dv = \int d\,uv - \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

Hints:

* whatever dv is, need to be able to find v

* helps if du is simpler than u and v is simpler than dv

* see chart pg. 530 (goal is to make simpler integral)

*choose u in this order:

Inverse Trig
Logs
Algebraic
Trig
Exponential

ILATE!

1) $\int x \cdot \cos x \, dx$

$\text{IL\textcircled{A}TE}$:

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$uv - \int v \, du$

$x \sin x - \int \sin x \, dx$

$x \sin x + \cos x + C$

2) $\int \ln x \, dx$

$\text{I\textcircled{L}ATE}$:

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$\ln x \cdot x - \int x \cdot \frac{1}{x} \, dx$

$x \ln x - \int 1 \, dx$

$x \ln x - x + C$

*3) $\int x^2 e^x \, dx$

$\text{IL\textcircled{A}TE}$:

$u = x^2 \qquad dv = e^x dx$

$du = 2x \, dx \qquad v = e^x$

$x^2 e^x - \int e^x \cdot 2x \, dx$

$x^2 e^x - 2 \int x e^x \, dx \longrightarrow$ $\text{IL\textcircled{A}TE}$:

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$x^2 e^x - 2\left(x e^x - \int e^x dx\right)$

$x^2 e^x - 2x e^x + 2e^x + C$

4) $\boxed{\int e^x \cos x \, dx}$

ILATE:

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$e^x \cos x - \int e^x \cdot -\sin x \, dx$

$e^x \cos x + \int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$e^x \cos x + e^x \sin x - \int e^x \cdot \cos x \, dx$

$e^x \cos x + e^x \sin x - \boxed{\int e^x \cos x \, dx}$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x}{2}$$

*5) $\int x e^x \, dx$

ILATE

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$x e^x - \int e^x dx$

$x e^x - e^x + C$

6) $\int x^2 \ln x \, dx$

IⓁATE

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$

$\ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$

$\frac{x^3}{3} \cdot \ln x - \frac{1}{3} \int x^2 dx$

$\frac{x^3}{3} \cdot \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$

$\frac{x^3}{3} \cdot \ln x - \frac{x^3}{9} + C$

$-c \; {}^{S} \; c$
$\quad {}_{-s}$

7) $\int x^2 \sin x \, dx$

ILⒶTE

$u = x^2 \quad dv = \sin x dx$

$du = 2x\, dx \quad v = -\cos x$

$x^2 \cdot -\cos x - \int -\cos x \cdot 2x \, dx$

$-x^2 \cos x + 2 \int x \cos x \, dx$

ILⒶTE

$u = x \quad dv = \cos x dx$

$du = dx \quad v = \sin x$

$-x^2 \cos x + 2\left(x \sin x - \int \sin x dx\right)$

$-x^2 \cos x + 2x \sin x + 2\cos x + C$

- Tabular Integration:

$$\int f(x) \cdot g(x)\, dx$$

- where $f(x)$ differentiates to 0 in several steps
- $g(x)$ integrates repeatedly

Steps:

1) make list of derivatives and integrals
2) alternate signs of u
3) link parts diagonally
4) combine together

Ex 1) $\int x e^x dx$ (compare to #5)

derive	u	dv	integrate
	x	e^x	
	-1	e^x	
	0	e^x	

$$xe^x - e^x + C$$

Ex 2) $\int x^2 e^x\,dx$ (compare to #3)

	u	dv
		e^x
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
	0	

$x^2e^x - 2xe^x + 2e^x + C$

Ex 3) $\int x^3 \sin x\,dx$

	u	dv
$+$	x^3	$\sin x$
$-$	$3x^2$	$-\cos x$
$+$	$6x$	$-\sin x$
$-$	6	$\cos x$
	0	$\sin x$

$-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

Ex 4) $\int x^2 e^{2x}\,dx$

	u	dv
		e^{2x}
$+$	x^2	$\frac{1}{2}e^{2x}$
$-$	$2x$	$\frac{1}{4}e^{2x}$
$+$	2	$\frac{1}{8}e^{2x}$
	0	

$e^{2x} \longrightarrow \int e^{2x}$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^{2x}$

$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$