

TRIG LIMITS:

$$* \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$* \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\begin{aligned} 1) \lim_{x \rightarrow 0} \frac{\tan x}{x} &= \frac{\frac{\sin x}{\cos x}}{x} = \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{x \cos x} \\ &= \left(\frac{\sin x}{x} \right) \cdot \frac{1}{\cos x} \\ &\quad \downarrow \\ &= 1 \cdot \frac{1}{\cos 0} = 1 \cdot 1 = \boxed{1} \end{aligned}$$

$$\begin{aligned} 2) \lim_{x \rightarrow 0} \frac{\sin 4x}{x} &= \frac{\sin 4x}{x} \cdot \frac{4}{4} = \frac{4 \sin 4x}{4x} = 4 \cdot 1 = \boxed{4} \end{aligned}$$

$$\begin{aligned} 3) \lim_{x \rightarrow 0} \frac{\tan^2 x}{x} &= \frac{\sin^2 x}{x \cos^2 x} \cdot \frac{x}{x} = \frac{\sin x \cdot \sin x \cdot x}{x \cdot \cos^2 x \cdot x} \\ &= \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{x}{\cos^2 x} = \frac{0}{1} \\ &= 1 \cdot 1 \cdot 0 = \boxed{0} \end{aligned}$$