6.2 Growth and Decay

* Rate of change proportional to amount: $\frac{dy}{dt} = Ky$
* Rate of change inversely proportional to amount: $\frac{dy}{dt} = \frac{K}{y}$

Proof: $\frac{dy}{dt} = Ky$

$\int \frac{1}{y}\,dy = \int K\,dt$

$\ln y = Kt + C$

$y = e^{Kt+C}$

$y = Ce^{Kt}$

$K > 0$ Growth

$K < 0$ Decay

(1) Rate of change of y is proportional to y. When [1] $t = 0$, $y = 2$. When [2] $t = 2$, $y = 4$. What is y when [3] $t = 3$?

[1] Find C

$\frac{dy}{dt} = Ky$

$\int \frac{1}{y}\,dy = \int K\,dt$

$\ln y = Kt + C$

$y = e^{Kt+C}$

$y = Ce^{Kt}$

$2 = Ce^{0}$

$C = 2$

$y = 2e^{Kt}$

[2] Find K

$4 = 2e^{2K}$

$2 = e^{2K}$

$\ln 2 = 2K$

$K = \frac{\ln 2}{2}$

$y = 2e^{\frac{\ln 2}{2}t}$

[3] Solve

$y = 2e^{\left(\frac{\ln 2}{2} \cdot 3\right)}$

$y \approx 5.657$