

Integrals: 5.7

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{1}{u \sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

*1)

$$\int \frac{2}{9 + 4x^2} dx$$

$a = 3$
 $u = 2x$
 $du = 2dx$

$$\frac{1}{3} \arctan \frac{u}{3} + C$$
$$= \int \frac{1}{3^2 + u^2} du = \frac{1}{3} \arctan \left(\frac{2x}{3} \right) + C$$

2)

$$\int \frac{3x}{\sqrt{1 - 9x^2}} dx$$

$u = 1 - 9x^2$
 $\frac{du}{-6} = \frac{-18x dx}{-6}$
 $-\frac{1}{6} du = 3x dx$

$$-\frac{1}{6} \int \frac{1}{\sqrt{u}} du$$
$$-\frac{1}{6} \int u^{-1/2} du$$
$$= -\frac{1}{6} \cdot 2 u^{1/2} = -\frac{1}{3} u^{1/2} = \frac{1}{3} (1 - 9x^2)^{1/2} + C$$