5.6 and 5.7 HW Solutions (Selected)

43) $g(x) = 3\arccos\left(\frac{1}{2}x\right)$

$$g'(x) = -3 \cdot \frac{1}{\sqrt{1-\left(\frac{1}{2}x\right)^2}} \cdot \frac{1}{2} = \frac{-3}{\sqrt{4-x^2}}$$

47) $g(x) = \dfrac{\arcsin 3x}{x}$

$$g'(x) = \frac{x\left(\frac{1}{\sqrt{1-(3x)^2}} \cdot 3\right) - \arcsin 3x}{x^2}$$

49) $h(t) = \sin(\arccos t)$

$$h'(t) = \cos(\arccos t) \cdot \frac{-1}{\sqrt{1-t^2}} = \frac{-t}{\sqrt{1-t^2}}$$

51) $y = x \cdot \arccos x - (1-x^2)^{1/2}$

$$y' = x \cdot \left(\frac{-1}{\sqrt{1-x^2}}\right) + \arccos x - \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$$

$$y' = \arccos x$$

5) $\displaystyle\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

$a=1$
$u=2x$
$du=2dx$
$\frac{1}{2}du=dx$

$\displaystyle\frac{1}{2}\int \frac{2}{2x\sqrt{(2x)^2-1}}\,dx$

$\displaystyle\int \frac{1}{u\sqrt{u^2-1}}\,du = \operatorname{arcsec}|2x| + c$

7) $\displaystyle\int \frac{x^3}{x^2-1}\,dx$

$$\begin{array}{r} x \\ x^2+1 \,\big)\, x^3 \quad 0x \\ -(x^3+x) \\ \hline -x \end{array}$$

$x - \frac{x}{x^2+1}$

$\displaystyle\int\left(x-\frac{x}{x^2+1}\right)dx = \int x\,dx - \int \frac{x}{x^2+1}\,dx$

$= \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + c$

13) $\displaystyle\int \frac{e^{2x}}{4+e^{4x}}\,dx$

$a=2$
$u=e^{2x}$
$du=2e^{2x}dx$
$\frac{1}{2}du=e^{2x}dx$

$\displaystyle\frac{1}{2}\int \frac{1}{2^2+u^2}\,du = \frac{1}{2}\cdot\frac{1}{2}\arctan\left(\frac{e^{2x}}{2}\right)+c$

$=\frac{1}{4}\arctan\left(\frac{e^{2x}}{2}\right)+c$

31) $\displaystyle\int_0^2 \frac{1}{x^2-2x+2}\,dx$

$x^2-2x+\boxed{1}+2-\boxed{1}$
$(x-1)^2+1$

$\displaystyle\int_0^2 \frac{1}{(x-1)^2+1}\,dx = \arctan(x-1)\Big|_0^2$

$\arctan 1 - \arctan(-1)$

$\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

33) $\int \frac{2x}{x^2+6x+13}\,dx$

$u = x^2+6x+13$

$du = (2x+6)\,dx$

$$\int \frac{2x+6-6}{x^2+6x+13}\,dx = \int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{1}{x^2+6x+13}\,dx$$

(u-sub) (complete □)

$x^2+6x+\boxed{9}+13-\boxed{9}$

$(x+3)^2+4$

$$= \ln|x^2+6x+13| - 6\int \frac{1}{(x+3)^2+4}\,dx$$

$$= \ln|x^2+6x+13| - 6\cdot\frac{1}{2}\arctan\left(\frac{x+3}{2}\right)+C$$

$$= \ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right)+C$$

41) $\int \frac{x}{x^4+2x^2+2}\,dx$

$x^4+2x^2+\boxed{1}+2-\boxed{1}$

$(x^2+1)^2+1$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{1}{u^2+1^2}\,du$$

$$= \frac{1}{2}\arctan\left(\frac{x^2+1}{1}\right)+C$$

$$= \frac{1}{2}\arctan(x^2+1)+C$$