5.6 - 5.7 Practice:

1) $y = \sin^{-1}(2x)$

$$y' = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

2) $y = \tan^{-1}(3x)$

$$y' = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$$

3) $y = \sec^{-1}(e^{2x})$

$$y' = \frac{1}{|e^{2x}| \cdot \sqrt{(e^{2x})^2 - 1}} \cdot 2e^{2x} = \frac{2\cancel{e^{2x}}}{\cancel{e^{2x}}\sqrt{e^{4x}-1}}$$

$$y' = \frac{2}{\sqrt{e^{4x}-1}}$$

4) $y = \frac{\arcsin(3x)}{x}$

$$y' = \frac{x\left(\frac{1}{\sqrt{1-(3x)^2}} \cdot 3\right) - \arcsin(3x)}{x^2}$$

$$y' = \frac{\frac{3x}{\sqrt{1-9x^2}} - \arcsin(3x)}{x^2}$$

5) $\int \frac{2}{9+4x^2}\,dx = \int \frac{2}{3^2+(2x)^2}\,dx$ $\quad a=3$, $u=2x$, $du=2dx$

$\int \frac{1}{3^2+u^2}\,du = \frac{1}{3}\arctan\left(\frac{u}{3}\right)+C$

$\frac{1}{3}\arctan\left(\frac{2x}{3}\right)+C$

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6) $\int \frac{3x}{\sqrt{1-9x^2}}\,dx$ $\quad u=1-9x^2$

$\frac{du}{-6} = \frac{-18x\,dx}{-6}$

$-\frac{1}{6}du = 3x\,dx$

$-\frac{1}{6}\int \frac{1}{\sqrt{u}}\,du$

$-\frac{1}{6}\int u^{-1/2}\,du = -\frac{1}{6}\cdot 2u^{1/2}+C$

$= -\frac{1}{3}u^{1/2}+C = -\frac{1}{3}\sqrt{1-9x^2}+C$

7) $\int \frac{x}{2x^2\sqrt{4x^4-36}}\,dx$ $\quad u=2x^2$, $a=6$, $du=4x\,dx$, $\frac{1}{4}du=x\,dx$

$\sqrt{(2x^2)^2-6^2}$

$\frac{1}{4}\int \frac{1}{u\sqrt{u^2-6^2}}\,du = \frac{1}{4}\cdot\frac{1}{6}\sec^{-1}\left|\frac{2x^2}{6}\right|+C$

$\frac{1}{24}\sec^{-1}\left(\frac{x^2}{3}\right)+C$

$$\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx \qquad a=2 \quad u=x \quad du=dx \qquad u(0)=0 \quad u(1)=1$$

$$\sqrt{2^2-x^2}$$

$$\int_0^1 \frac{1}{\sqrt{2^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{2}\right)\Big|_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

9) $$\int_{\sqrt{3}}^{3} \frac{4}{9+x^2}\,dx = \int_{\sqrt{3}}^{3} \frac{4}{3^2+x^2}\,dx \qquad a=3 \quad u=x \quad du=dx$$

$$4\cdot\int_{\sqrt{3}}^{3} \frac{1}{3^2+u^2}\,du = 4\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right)\Big|_{\sqrt{3}}^{3}$$

$$= \frac{4}{3}\left[\tan^{-1}(1) - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right]$$

$$\frac{4}{3}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{4}{3}\left(\frac{\pi}{12}\right) = \frac{\pi}{9}$$

$$\frac{3\pi}{12} - \frac{2\pi}{12}$$

10) $\int \frac{dx}{x^2+4x+13} = \int \frac{1}{(x+2)^2+9}\,dx$ $\quad a=3$, $u=x+2$, $du=dx$

$x^2+4x+\boxed{4}+13-\boxed{4}$

$(x+2)^2+9$

$$\int \frac{1}{3^2+u^2}\,du = \frac{1}{3}\arctan\left(\frac{x+2}{3}\right)+C$$

11) $\int \frac{dx}{x^2+6x+15} = \int \frac{1}{(x+3)^2+6}\,dx$ $\quad a=\sqrt{6}$, $u=x+3$, $du=dx$

$x^2+6x+\boxed{9}+15-\boxed{9}$

$(x+3)^2+6$

$$\int \frac{1}{(\sqrt{6})^2+u^2}\,du = \frac{1}{\sqrt{6}}\cdot\arctan\left(\frac{x+3}{\sqrt{6}}\right)+C$$

12) $\int_0^{\frac{\pi}{6}} \frac{dx}{4x^2+7}$ $\quad a=\sqrt{7}$, $u=2x$, $du=2dx$, $\frac{1}{2}du=dx$

$(2x)^2+(\sqrt{7})^2$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{7}}\left(\tan^{-1}\left(\frac{2x}{\sqrt{7}}\right)\right)\Bigg|_0^{\pi/6} = \frac{1}{2\sqrt{7}}\left(\tan^{-1}\left(\frac{\pi}{3\sqrt{7}}\right)-\tan^{-1}(0)\right)$$

$$=\frac{1}{2\sqrt{7}}\left(\tan^{-1}\left(\frac{\pi}{3\sqrt{7}}\right)\right)$$

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13) $\int \frac{1}{x+7}\,dx$

$u = x+7$

$du = dx$

$$\int \frac{1}{u}\,du = \ln|u| + C = \ln|x+7| + C$$

14) $4\int \frac{1}{\sqrt{64-16x^2}}\,dx$

$\sqrt{8^2-(4x)^2}$

$a = 8$

$u = 4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$$\cancel{\tfrac{1}{4}} \cdot \cancel{4}\int \frac{1}{\sqrt{8^2-u^2}}\,du$$

$$= \sin^{-1}\left(\frac{4x}{8}\right) + C = \sin^{-1}\left(\frac{x}{2}\right) + C$$

15) $\int \frac{\tan x}{\sin^2 x\sqrt{\cot^2 x - 16}}\,dx$

$\sqrt{(\cot x)^2 - 4^2}$

$a = 4$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$$= -\int \frac{\frac{1}{u}}{\sqrt{u^2-4^2}}\,du = -\int \frac{1}{u\sqrt{u^2-4^2}}\,du$$

$$= -\frac{1}{4}\sec^{-1}\left|\frac{\cot x}{4}\right| + C$$

16) $y = \sqrt{\tan^{-1}x} = (\tan^{-1}x)^{1/2}$

$$y' = \frac{1}{2}(\tan^{-1}x)^{-1/2} \cdot \frac{1}{1+x^2}$$

$$y' = \frac{1}{2(1+x^2)(\sqrt{\tan^{-1}x})}$$

17) $y = \sin^{-1}(2x+1)$

$$y' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2 = \frac{2}{\sqrt{1-(2x+1)^2}}$$

18) $y = \arccos(e^{2x})$

$$y' = \frac{-1}{\sqrt{1-(e^{2x})^2}} \cdot 2e^{2x} = \frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$$

19) $y = x \cdot \ln(\arctan x)$

$$y' = x \cdot \frac{1}{\arctan x} \cdot \frac{1}{1+x^2}$$

$$+ \ln(\arctan x) \cdot 1$$

$$= \frac{x}{(1+x^2)\arctan x} + \ln(\arctan x)$$