

5.6 - 5.7 Practice:

$$1) y = \sin^{-1}(2x)$$

$$y' = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

$$2) y = \tan^{-1}(3x)$$

$$y' = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$$

$$3) y = \sec^{-1}(e^{2x})$$

$$y' = \frac{1}{|e^{2x}| \cdot \sqrt{(e^{2x})^2 - 1}} \cdot 2e^{2x} = \frac{2e^{2x}}{e^{2x} \sqrt{e^{4x} - 1}}$$

$$y' = \frac{2}{\sqrt{e^{4x} - 1}}$$

$$4) y = \frac{\arcsin(3x)}{x}$$

$$y' = \frac{x \left(\frac{1}{\sqrt{1-(3x)^2}} \cdot 3 \right) - \arcsin(3x)}{x^2}$$

$$y' = \frac{3x}{\sqrt{1-9x^2}} - \frac{\arcsin(3x)}{x^2}$$

$$5) \int \frac{2}{9+4x^2} dx = \int \frac{2}{3^2+(2x)^2} dx \quad \begin{array}{l} a=3 \\ u=2x \\ du=2dx \end{array}$$

$$\int \frac{1}{3^2+u^2} du = \frac{1}{3} \arctan\left(\frac{u}{3}\right) + C$$

$$\frac{1}{3} \arctan\left(\frac{2x}{3}\right) + C$$

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$$6) \int \frac{3x}{\sqrt{1-9x^2}} dx \quad \begin{array}{l} u=1-9x^2 \\ du = -18x dx \\ \frac{du}{-6} = \frac{-18x dx}{-6} \\ -\frac{1}{6} du = 3x dx \end{array}$$

$$-\frac{1}{6} \int \frac{1}{\sqrt{u}} du$$

$$-\frac{1}{6} \int u^{-1/2} du = -\frac{1}{6} \cdot 2 u^{1/2} + C$$

$$= -\frac{1}{3} u^{1/2} + C = -\frac{1}{3} \sqrt{1-9x^2} + C$$

$$7) \int \frac{x}{2x^2 \sqrt{4x^4-36}} dx$$

$$\frac{\sqrt{(2x^2)^2-6^2}}{\sqrt{(2x^2)^2-6^2}}$$

$$\begin{array}{l} u=2x^2 \\ a=6 \\ du=4x dx \\ \frac{1}{4} du = x dx \end{array}$$

$$\frac{1}{4} \int \frac{1}{u \sqrt{u^2-6^2}} du = \frac{1}{4} \cdot \frac{1}{6} \sec^{-1} \left| \frac{2x^2}{6} \right| + C$$

$$\frac{1}{24} \sec^{-1} \left(\frac{x^2}{3} \right) + C$$

STAP

$$\int_0^1 \frac{1}{\sqrt{4-x^2}} dx \quad \begin{array}{l} a=2 \\ u=x \\ du=dx \end{array} \quad \begin{array}{l} u(0)=0 \\ u(1)=1 \end{array}$$

$$\int_0^1 \frac{1}{\sqrt{2^2-x^2}}$$

$$\int_0^1 \frac{1}{\sqrt{2^2-u^2}} du = \sin^{-1}\left(\frac{u}{2}\right) \Big|_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

9) $\int_{\sqrt{3}}^3 \frac{4}{9+x^2} dx = \int_{\sqrt{3}}^3 \frac{4}{3^2+x^2} dx \quad \begin{array}{l} a=3 \\ u=x \\ du=dx \end{array}$

4. $\int_{\sqrt{3}}^3 \frac{1}{3^2+u^2} du = 4 \cdot \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) \Big|_{\sqrt{3}}^3$

$$= \frac{4}{3} \left[\tan^{-1}(1) - \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \right]$$

$$\frac{4}{3} \left(\frac{\pi}{4} - \frac{\pi}{6} \right) = \frac{4}{3} \left(\frac{\pi}{12} \right) = \frac{\pi}{9}$$

$$\frac{3\pi}{12} - \frac{2\pi}{12}$$

$$10) \int \frac{dx}{x^2+4x+13} = \int \frac{1}{(x+2)^2+9} dx \quad \begin{array}{l} a=3 \\ u=x+2 \\ du=dx \end{array}$$

$$x^2+4x+\boxed{4}+13-\boxed{4}$$

$$(x+2)^2+9$$

$$\int \frac{1}{3^2+u^2} du = \frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + C$$

$$11) \int \frac{dx}{x^2+6x+15} = \int \frac{1}{(x+3)^2+6} dx \quad \begin{array}{l} a=\sqrt{6} \\ u=x+3 \\ du=dx \end{array}$$

$$x^2+6x+\boxed{9}+15-\boxed{9}$$

$$(x+3)^2+6$$

$$\int \frac{1}{(\sqrt{6})^2+u^2} du = \frac{1}{\sqrt{6}} \cdot \arctan\left(\frac{x+3}{\sqrt{6}}\right) + C$$

$$(12) \int_0^{\pi/6} \frac{dx}{4x^2+7} \quad \begin{array}{l} a=\sqrt{7} \\ u=2x \\ du=2dx \\ \frac{1}{2}du=dx \end{array}$$

$$(2x)^2+(\sqrt{7})^2$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{7}} \left(\tan^{-1}\left(\frac{2x}{\sqrt{7}}\right) \right) \Big|_0^{\pi/6} = \frac{1}{2\sqrt{7}} \left(\tan^{-1}\left(\frac{\pi}{3\sqrt{7}}\right) - \tan^{-1}(0) \right)$$

$$= \frac{1}{2\sqrt{7}} \left(\tan^{-1}\left(\frac{\pi}{3\sqrt{7}}\right) \right)$$

STAPLE

$$\int \frac{1}{x+7} dx \quad u=x+7$$

$$du=dx$$

$$\int \frac{1}{u} du = \ln|u| + C = \ln|x+7| + C$$

$$(14) \quad 4 \int \frac{1}{\sqrt{64-16x^2}} dx \quad a=8$$

$$u=4x$$

$$du=4dx$$

$$\sqrt{8^2-(4x)^2}$$

$$\frac{1}{4} du = dx$$

$$\frac{1}{4} \int \frac{1}{\sqrt{8^2-u^2}} du$$

$$= \sin^{-1}\left(\frac{4x}{8}\right) + C = \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$(15) \quad \int \frac{\tan x}{\sin^2 x \sqrt{\cot^2 x - 16}} dx \quad a=4$$

$$u = \cot x$$

$$du = -\csc^2 x dx$$

$$-du = \csc^2 x dx$$

$$\sqrt{(\cot x)^2 - 4^2}$$

$$= - \int \frac{\frac{1}{u}}{\sqrt{u^2 - 4^2}} du = - \int \frac{1}{u \sqrt{u^2 - 4^2}} du$$

$$= - \frac{1}{4} \sec^{-1} \left| \frac{\cot x}{4} \right| + C$$

$$(16) \quad y = \sqrt{\tan^{-1} x} = (\tan^{-1} x)^{1/2}$$

$$y' = \frac{1}{2} (\tan^{-1} x)^{-1/2} \cdot \frac{1}{1+x^2}$$

$$y' = \frac{1}{2(1+x^2)(\sqrt{\tan^{-1} x})}$$

$$17) y = \sin^{-1}(2x+1)$$

$$y' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2 = \frac{2}{\sqrt{1-(2x+1)^2}}$$

$$18) y = \arccos(e^{2x})$$

$$y' = \frac{-1}{\sqrt{1-(e^{2x})^2}} \cdot 2e^{2x} = \frac{-2e^{2x}}{\sqrt{1-e^{4x}}}$$

$$19) y = x \cdot \ln(\arctan x)$$

$$y' = x \cdot \frac{1}{\arctan x} \cdot \frac{1}{1+x^2}$$

$$+ \ln(\arctan x) \cdot 1$$

$$= \frac{x}{(1+x^2)\arctan x} + \ln(\arctan x)$$