7) Find particular solution that satisfies

$f''(x) = \sin x + e^{2x}$

$f'(0) = \frac{1}{2}$

$f(0) = \frac{1}{4}$

$\int e^{2x}\,dx \quad u = 2x \quad du = 2\,dx \quad \frac{1}{2}du = dx$

$= \frac{1}{2}e^{2x}$

$f'(x) = \int (\sin x + e^{2x})\,dx$

$f'(x) = -\cos x + \frac{1}{2}e^{2x} + C$

$\frac{1}{2} = -\cos 0 + \frac{1}{2}e^{0} + C$

$\frac{1}{2} = -1 + \frac{1}{2} + C$

$C = 1$

$f'(x) = -\cos x + \frac{1}{2}e^{2x} + 1$

$f(x) = \int (-\cos x + \frac{1}{2}e^{2x} + 1)\,dx$

$f(x) = -\sin x + \frac{1}{4}e^{2x} + x + C$

$\frac{1}{4} = -\sin 0 + \frac{1}{4}e^{0} + 0 + C$

$\frac{1}{4} = \frac{1}{4} + C$

$C = 0$

$f(x) = -\sin x + \frac{1}{4}e^{2x} + x$