

Function	1 <sup>st</sup> Derivative/Critical #s	Intervals of increasing/decreasing	2 <sup>nd</sup> Derivative/Possible POI	Concavity
$x^3 - 3x + 1$				
$y = x^2 - x - 1$				
$y = 2x^4 - 4x^2 + 1$				
$y = xe^x$				
$y = -2x^3 + 6x^2 - 3$				
$y = \frac{x}{x-1}$				
$y = \sin x + \cos x$				

### 3.1-3.4 Review Key

$$x^3 - 3x + 1$$

$$\begin{aligned} f'(x) &= 3x^2 - 3 = 0 \\ 3(x^2 - 1) &= 0 \\ 3(x+1)(x-1) &= 0 \\ x &= -1, 1 \end{aligned}$$



Inc:  $(-\infty, -1), (1, \infty)$

Dec:  $(-1, 1)$

$$\begin{aligned} f''(x) &= 6x = 0 \\ x &= 0 \end{aligned}$$

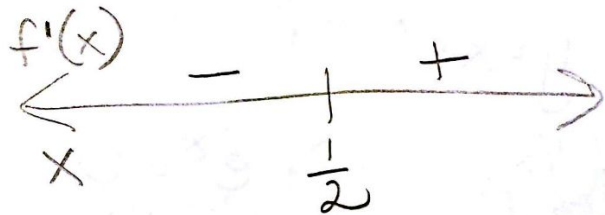


CC $\uparrow$ :  $(0, \infty)$

CC $\downarrow$ :  $(-\infty, 0)$

$$2) \quad y = x^2 - x - 1$$

$$\begin{aligned} f'(x) &= 2x - 1 = 0 \\ x &= \frac{1}{2} \end{aligned}$$



Inc:  $(\frac{1}{2}, \infty)$

Dec:  $(-\infty, \frac{1}{2})$

$$f''(x) = 2$$

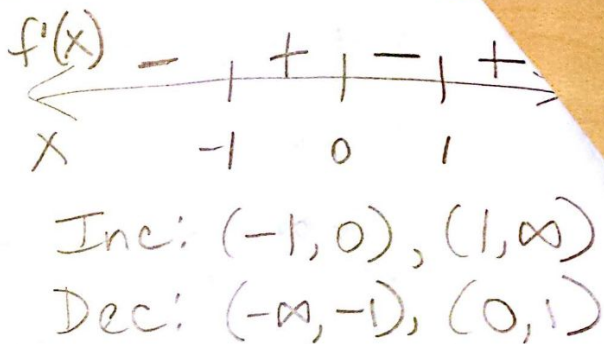
CC $\uparrow$ :  $(-\infty, \infty)$

$$3) \quad 2x^4 - 4x^2 + 1$$

$$f'(x) = 8x^3 - 8x = 0$$

$$8x(x^2 - 1) = 0$$

$$x = 0, -1, 1$$



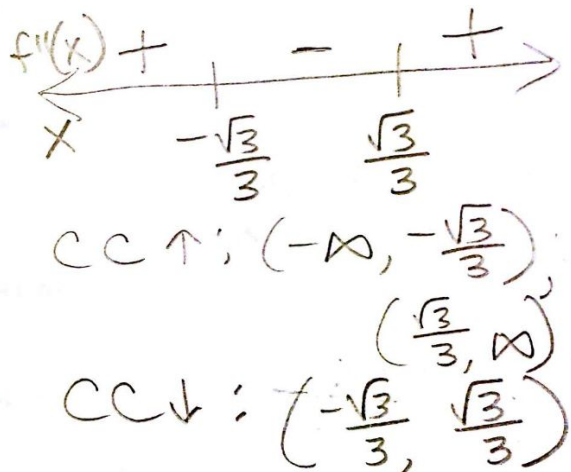
$$f''(x) = 24x^2 - 8 = 0$$

$$8(3x^2 - 1) = 0$$

$$3x^2 = \frac{1}{3}$$

$$x^2 = \frac{1}{9}$$

$$x = \pm \frac{\sqrt{3}}{3}$$

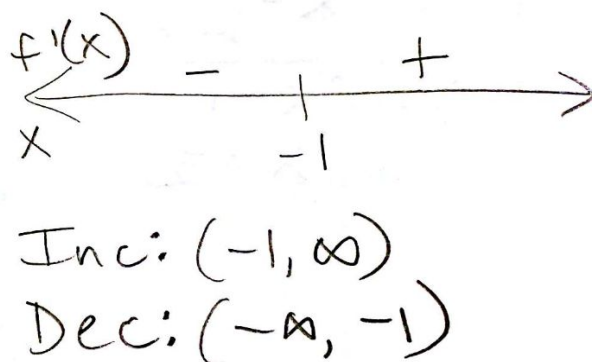


$$4) \quad y = xe^x$$

$$f'(x) = xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$$x = -1$$

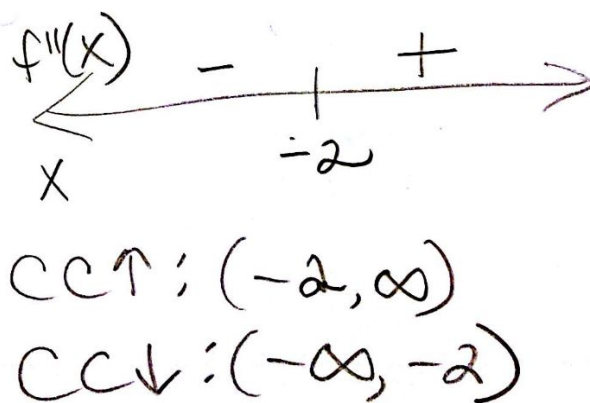


$$f''(x) = (xe^x + e^x) + e^x$$

$$= xe^x + 2e^x$$

$$= e^x(x+2) = 0$$

$$x = -2$$





$$y = -2x^3 + 6x^2 - 3$$

$$y' = -6x^2 + 12x = 0$$

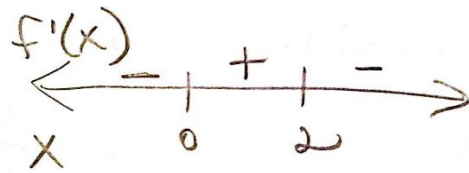
$$-6x(x-2) = 0$$

$$x = 0, 2$$

$$y'' = -12x + 12 = 0$$

$$-12(x-1) = 0$$

$$x = 1$$



Inc:  $(0, 2)$

Dec:  $(-\infty, 0), (2, \infty)$



CC $\uparrow$ :  $(-\infty, 1)$

CC $\downarrow$ :  $(1, \infty)$

6)  $y = \frac{x}{x-1}$

$$y' = \frac{(x-1) - x(1)}{(x-1)^2}$$

$$y' = \frac{x-1-x}{(x-1)^2}$$

$$(x-1)^2 = 0$$

$$x = 1$$

$$y' = -1(x-1)^{-2}$$

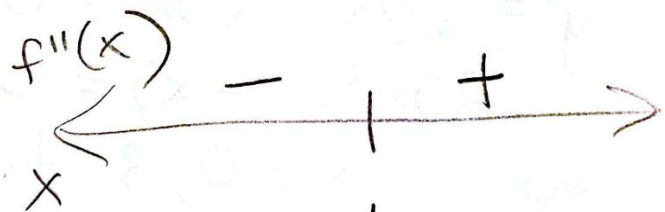
$$y'' = 2(x-1)^{-3}$$

$$y'' = \frac{2}{(x-1)^3} \quad x = 1$$



Inc: none

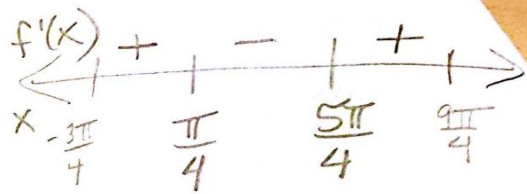
Dec:  $(-\infty, 1), (1, \infty)$



CC $\uparrow$ :  $(1, \infty)$

CC $\downarrow$ :  $(-\infty, 1)$

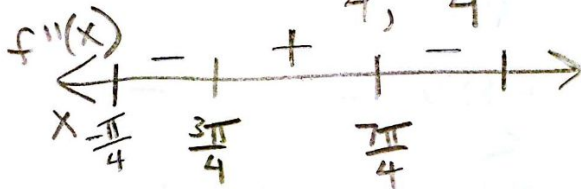
$$\begin{aligned} 7) \quad y &= \sin x + \cos x \\ y' &= \cos x - \sin x = 0 \\ \cos x &= \sin x \\ x &= \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$



$$\text{Inc: } \left(-\frac{3\pi}{4} + 2\pi k, \frac{\pi}{4} + 2\pi k\right) \\ k \in \mathbb{Z}$$

$$\text{Dec: } \left(\frac{\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k\right)$$

$$\begin{aligned} y'' &= -\sin x - \cos x = 0 \\ -\cos x &= \sin x \\ x &= \frac{3\pi}{4}, \frac{7\pi}{4} \end{aligned}$$



$$\text{CC}\uparrow: \left(\frac{3\pi}{4} + 2\pi k, \frac{7\pi}{4} + 2\pi k\right)$$

$$\text{CC}\downarrow: \left(-\frac{\pi}{4} + 2\pi k, \frac{3\pi}{4} + 2\pi k\right)$$

$$k \in \mathbb{Z}$$

MC

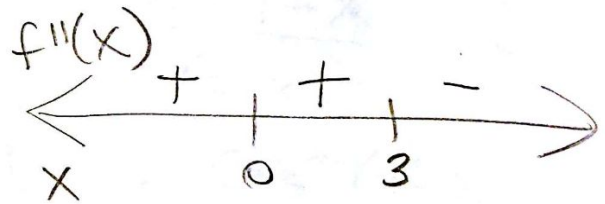
$$1) \quad y = 5x^4 - x^5$$

$$y' = 20x^3 - 5x^4$$

$$y'' = 60x^2 - 20x^3 = 0$$

$$20x^2(3-x) = 0$$

$$x = 0, 3$$



$$(3, 162)$$

$$y = x^{1/2}$$

$$y' = \frac{1}{2}x^{-1/2}$$

$$y' = \frac{1}{2\sqrt{x}}$$

$$(0,0), (4,2)$$

$$\frac{2-0}{4-0} = \frac{1}{2}$$

(B)

$$\frac{1}{2\sqrt{x}} = \frac{1}{2}$$

$$x=1 \quad (1,1)$$

3)  $f(x) = \frac{1}{3}x^4 - \frac{1}{5}x^5$

(C)

$$f'(x) = \frac{4}{3}x^3 - x^4$$

$$f''(x) = 4x^2 - 4x^3 = 0$$

$$4x^2(1-x) = 0$$

$$x=0, 1$$



4)  $y = x^3 + 3x^2 + 2$

$$y' = 3x^2 + 6x$$

$$y'' = 6x + 6 = 0$$

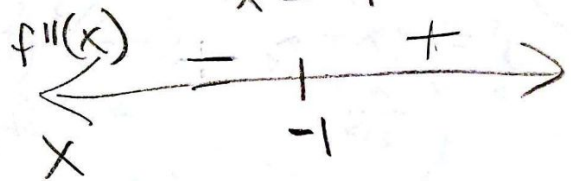
$$6(x+1) = 0$$

$$x = -1$$

(B)

$$f'(-1) = 3(-1)^2 + 6(-1)$$

$$f'(-1) = 3 - 6 = -3$$



$$f(-1) = -1 + 3 + 2 = 4$$

$$y - 4 = -3(x + 1)$$

$$y - 4 = -3x - 3$$

$$y = -3x + 1$$



$$5) y = \frac{-5}{x-2} = -5(x-2)^{-1}$$

$$y' = 5(x-2)^{-2} = \frac{5}{(x-2)^2}$$

$$y'' = \frac{-10}{(x-2)^3}$$



(E)

$$x-2=0 \\ x=2$$

$$6) f(x) = x^3 - 3x^2 + 12$$

(E)

$$f'(x) = 3x^2 - 6x = 0 \\ 3x(x-2) = 0 \\ x = 0, 2$$

x	y
-2	-8
0	12
2	8
4	28

abs min

$$7) y = (x+2)^5 (x-3)^4$$

(C)

$$y' = (x+2)^5 \cdot (4(x-3)^3) + (x-3)^4 (5(x+2)^4) = 0$$

$$y' = (x+2)^4 (x-3)^3 (4(x+2) + 5(x-3)) = 0 \\ (4x+8 + 5x-15) \\ (9x-7)$$

$$x = -2$$

$$x = 3$$

$$x = \frac{7}{9}$$

R:

a)  $f(x) = x^{1/3} + 4x^{1/3} \quad -8 \leq x \leq 8$

$$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} \quad (1)$$

$$= \frac{4}{3}x^{-2/3}(x+1) = \frac{4}{3}\left(\frac{x+1}{x^{2/3}}\right)$$

$f'(x) = 0$  at  $x = -1$

HTL:  $(-1, -3) \quad (1)$   $\frac{x^{2/3}}{x^{4/3}} = \frac{4x^{1/3}}{3} + \frac{4}{3x^{2/3}}$

b) VTL:  $(0, 0) \quad (1)$

$$\frac{4x+4}{3x^{2/3}} = \frac{4(x+1)}{3x^{2/3}}$$

c)

x	y
-8	8
-1	-3
0	0
8	24

Abs max:  $(8, 24) \quad (1)$

Abs min:  $(-1, -3) \quad (1)$

$$\frac{x^{2/3}}{x^{3/3}} = \frac{4}{9x^{2/3}} - \frac{8}{9x^{5/3}}$$

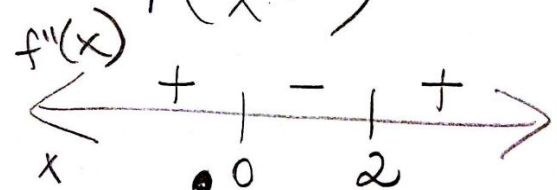
$$\frac{4x-8}{9x^{5/3}}$$

d)  $f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} \quad (1)$

$$f''(x) = \frac{4}{9}x^{-5/3}(x-2) = \frac{4}{9}\left(\frac{x-2}{x^{5/3}}\right)$$

CC ↓ :

$(1) (0, 2)$



e)

