1. The asymptotes of the graph of the parametric equations $x=\frac{1}{t}$, $y=\frac{t}{t+1}$ are

(A) $x=0,\ y=0$ (B) $x=0$ only (C) $x=-1,\ y=0$
(D) $x=-1$ only (E) $x=0,\ y=1$

2. The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi}\sqrt{3+\cos\theta}\,d\theta$ (B) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$ (C) $2\int_0^{\pi/2}(3+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ (E) $2\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$

3. Which of the following integrals gives the length of the graph of $y=\tan x$ between $x=a$ and $x=b$, where $0<a<b<\frac{\pi}{2}$?

(A) $\int_a^b\sqrt{x^2+\tan^2 x}\,dx$

(B) $\int_a^b\sqrt{x+\tan x}\,dx$

(C) $\int_a^b\sqrt{1+\sec^2 x}\,dx$

(D) $\int_a^b\sqrt{1+\tan^2 x}\,dx$

(E) $\int_a^b\sqrt{1+\sec^4 x}\,dx$

4. The length of the curve $y=\ln\sec x$ from $x=0$ to $x=b$, where $0<b<\frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b\sec x\,dx$

(B) $\int_0^b\sec^2 x\,dx$

(C) $\int_0^b(\sec x\tan x)\,dx$

(D) $\int_0^b\sqrt{1+(\ln\sec x)^2}\,dx$

(E) $\int_0^b\sqrt{1+\left(\sec^2 x\tan^2 x\right)}\,dx$

5. If $x=t^2-1$ and $y=2e^t$, then $\frac{dy}{dx}=$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

6. The area of the region enclosed by the polar curve $r=1-\cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

7. A particle moves in the xy-plane so that at any time t its coordinates are $x=t^2-1$ and $y=t^4-2t^3$. At $t=1$, its acceleration vector is

(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$

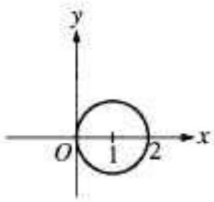
8. The area of the region enclosed by the polar curve $r=\sin(2\theta)$ for $0\le\theta\le\frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

9. If $x=t^3-t$ and $y=\sqrt{3t+1}$, then $\frac{dy}{dx}$ at $t=1$ is

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{8}{3}$ (E) 8

10. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

(A)


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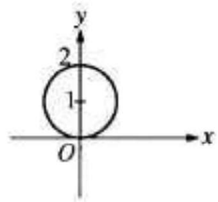

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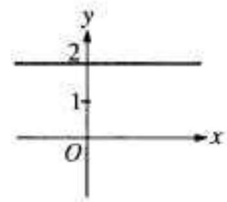
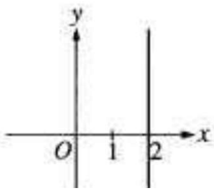
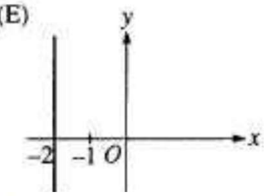


(D)


(E)


11. If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{3}{4t}$ (B) $\frac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\frac{3}{2}$

12. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

(A) $\int_0^4 \sqrt{4t+1}\, dt$

(B) $2\int_0^4 \sqrt{t^2+1}\, dt$

(C) $\int_0^4 \sqrt{2t^2+1}\, dt$

(D) $\int_0^4 \sqrt{4t^2+1}\, dt$

(E) $2\pi\int_0^4 \sqrt{4t^2+1}\, dt$

13. Consider the curve in the xy-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

(A) 20.086 (B) 0.342 (C) −0.005 (D) −0.011 (E) −0.033

14. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2+2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

(A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(-\frac{5}{16}, 4\right)$

15. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

16. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \leq t \leq \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

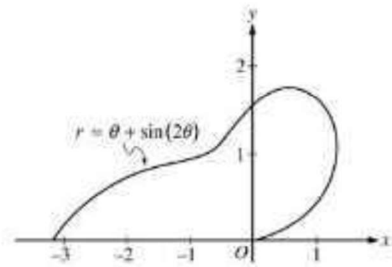
(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

1.	2.	3.	4.	5.	6.	7.	8.
9.	10.	11.	12.	13.	14.	15.	16.



The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

a.
b.
c.
d.

Problem Set A ANSWERS:1969BC#1,9,43 1973BC#10,14,40, 1985BC#4,24,30, 1993BC#5,6,23,25, 1997BC#2

1. C	2. D	3. E	4. A	5. A	6. C	7. D	8. D
9. B	10. D	A	12. D	13. D	14. A	15. E	16. D

(a) Area $= \frac{1}{2}\int_0^{\pi} r^2\, d\theta$

$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2\, d\theta = 4.382$

$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) $-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$

$\theta = 2.786$

$2: \begin{cases} 1: \text{equation} \\ 1: \text{answer} \end{cases}$

(c) Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

$2: \begin{cases} 1: \text{information about } r \\ 1: \text{information about the curve} \end{cases}$

(d) The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

$2: \begin{cases} 1: \theta = \frac{\pi}{3} \text{ or } 1.047 \\ 1: \text{answer with justification} \end{cases}$