

Practice Problems

Limit as x approaches infinity

$$1. \lim_{x \rightarrow \infty} \left(\frac{3x-7}{5x^4-8x+12} \right) =$$

$$2. \lim_{x \rightarrow \infty} \left(\frac{3x^4-2}{5x^4-2x+1} \right) =$$

$$3. \lim_{x \rightarrow \infty} \left(\frac{x^6-2}{10x^4-9x+8} \right) =$$

$$4. \lim_{x \rightarrow \infty} \left(\frac{7x^4-2}{5-2x^3-14x^4} \right) =$$

$$5. \lim_{x \rightarrow \infty} \left(\frac{\sin x}{e^x} \right) =$$

$$6. \lim_{x \rightarrow -\infty} \left(\frac{\sqrt{x^2-9}}{2x-3} \right) =$$

$$7. \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x^2-9}}{2x-3} \right) =$$

Limit as x approaches a number

$$8. \lim_{x \rightarrow 2} (x^3 - x + 1)$$

$$9. \lim_{x \rightarrow 2} \left(\frac{x^2-4}{x-2} \right) =$$

$$10. \lim_{x \rightarrow 2^+} \left(\frac{3}{x-2} \right) =$$

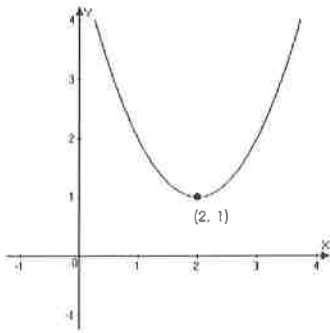
$$11. \lim_{x \rightarrow 2^+} \left(\frac{3}{x-2} \right) =$$

$$12. \lim_{x \rightarrow 2} \left(\frac{3}{x-2} \right) =$$

$$13. \lim_{x \rightarrow 2^+} \left(\frac{3}{2-x} \right) =$$

$$14. \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\sin x}{x} \right) =$$

$$15. \lim_{x \rightarrow \frac{\pi}{4}} \left(\frac{\tan x}{x} \right) =$$



4. The graph of $y = f(x)$ is shown above. $\lim_{x \rightarrow 2} ((f(x)^3) - 3f(x) + 7) =$

- (A) 1 (B) 5 (C) 7 (D) 9 (E) Does not exist

5. If $f(x) = \begin{cases} x^2 - 3x - 4, & x \neq -1 \\ 2, & x = -1 \end{cases}$, what is $\lim_{x \rightarrow -1} f(x)$?

- (A) -5 (B) 0 (C) 2 (D) 3 (E) Does not exist

6. $\lim_{x \rightarrow \infty} \left(\frac{2x^6 - 5x^3 + 10}{20 - 4x^2 - x^6} \right) =$

- (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2 (E) Does not exist

7. $\lim_{x \rightarrow \infty} \left(\frac{2x^5 - 5x^3 + 10}{20 - 4x^2 - x^6} \right) =$

- (A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

8. $\lim_{x \rightarrow \infty} \left(1 + e^{\frac{1}{2} + \frac{1}{x}} \right) =$

(A) $-\infty$ (B) 0 (C) $e^{\frac{1}{2}}$

(D) $1 + e^{\frac{1}{2}}$ (E) ∞

9. $\lim_{x \rightarrow 3^+} \frac{5}{3-x} =$

(A) $-\infty$ (B) -5 (C) 0

(D) $\frac{5}{3}$ (E) ∞

10. If $\lim_{n \rightarrow \infty} \left(\frac{5n^3}{20-3n-kn^3} \right) = \frac{1}{2}$, then $k =$

(A) -10 (B) -4 (C) $\frac{1}{4}$ (D) 4 (E) 10

11. Which of the following is/are true about the function g if $g(x) = \frac{(x-2)^2}{x^2+x-6}$?

- I. g is continuous at $x = 2$
- II. The graph of g has a vertical asymptote at $x = -3$
- III. The graph of g has a horizontal asymptote at $y = 0$

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

$$12. f(x) = \begin{cases} \sin x, & x < \frac{\pi}{4} \\ \cos x, & x > \frac{\pi}{4} \\ \tan x, & x = \frac{\pi}{4} \end{cases}$$

What is $\lim_{x \rightarrow \frac{\pi}{4}} f(x)$?

- (A) $-\infty$ (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) ∞

$$13. \lim_{x \rightarrow a} \left(\frac{\sqrt{x} - \sqrt{a}}{x - a} \right) =$$

- (A) $\frac{1}{2\sqrt{a}}$ (B) $\frac{1}{\sqrt{a}}$ (C) \sqrt{a} (D) $2\sqrt{a}$ (E) Does not exist

$$14. \lim_{x \rightarrow 0^+} \frac{\ln 2x}{2x} =$$

- (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

Free Response 2 (No calculator)

Given the function $f(x) = \frac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

- (a) What are the zeros of $f(x)$?
- (b) What are the vertical asymptotes of $f(x)$?
- (c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?
- (d) What is $\lim_{x \rightarrow \infty} f(x)$? What is $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$?

Answers to Unit 1 – Limit and Continuity Review

Practice Problems:

- | | |
|-----------------------------|-------|
| 1. 0 | |
| 2. $\frac{3}{5}$ | |
| 3. ∞ | |
| 4. $-\frac{1}{2}$ | |
| 5. 0 | |
| 6. $-\frac{1}{2}$ | |
| 7. $\frac{1}{2}$ | |
| 8. 7 | |
| 9. 4 | 5. A |
| 10. $-\infty$ | 6. A |
| 11. ∞ | 7. C |
| 12. does not exist | 8. D |
| 13. $-\infty$ | 9. A |
| 14. $\frac{2\sqrt{2}}{\pi}$ | 10. A |
| 15. $\frac{4}{\pi}$ | 11. B |
| | 12. D |
| | 13. A |
| | 14. A |

Free Response 2 (No calculator)

Given the function $f(x) = \frac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$.

- What are the zeros of $f(x)$?
- What are the vertical asymptotes of $f(x)$?
- The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?
- What is $\lim_{x \rightarrow \infty} f(x)$? What is $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$?

(a) The zeros of the function, $f(x)$, occur at $x = -3, 0, 1$

3 pts, 1 for each zero

(b) There is a vertical asymptote at $x = -2$

1 pt for the vertical asymptote

(c) $g(x) = \frac{1}{3}x$

2 pts for $g(x)$

(d) $\lim_{x \rightarrow \infty} f(x) = \infty$

1 pt $\lim_{x \rightarrow \infty} f(x)$

$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$

2 pts for $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$