PRACTICE PROBLEMS

All of these problems are intended to be worked without a calculator.

1. $\int \cos 3x \, dx =$

(A) $-3\sin 3x + C$ (B) $-\sin 3x + C$ (C) $-\frac{1}{3}\sin 3x + C$

(D) $\frac{1}{3}\sin 3x + C$ (E) $3\sin 3x + C$

2. $\int \frac{1-3y}{\sqrt{2y-3y^2}} \, dy =$

(A) $4\sqrt{2y-3y^2} + C$ (B) $2\sqrt{2y-3y^2} + C$

(C) $\frac{1}{2}\ln\left(\sqrt{2y-3y^2}\right) + C$ (D) $\frac{1}{4}\ln\left(\sqrt{2y-3y^2}\right) + C$

(E) $\sqrt{2y-3y^2} + \mathrm{C}$

3. $\int \frac{dy}{\sqrt{1-4y^2}} =$

(A) $-\frac{1}{2}\sqrt{1-4y^2} + C$ (B) $\frac{1}{2}\sqrt{1-4y^2} + C$

(C) $\sin^{-1} 2y + C$ (D) $\frac{1}{2}\sin^{-1} 2y + C$

(E) $-\frac{1}{2}\sin^{-1} 2y + C$

4. $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 3x} =$

(A) -3 (B) -1 (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$ (E) 3

5. $\int_e^{e^3} \frac{\ln x}{x}\, dx =$

(A) 2 (B) $\frac{5}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 8

6. $\int_1^4 |x-3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

For 7 – 20, evaluate the integral. For each integral, assume a domain on which the integral is defined. Remember – no calculator!

7. $\int \frac{x+1}{x^2-1}\,dx$

8. $\int \frac{e^x}{1+e^x}\,dx$

9. $\int \frac{e^x}{1+e^{2x}}\,dx$

10. $\int \left(x-\frac{1}{2x}\right)^2 dx$

11. $\int (x+2)\sqrt{x-3}\,dx$

12. $\int \sqrt{4-2x}\,dx$

13. $\int \frac{\cos(x-1)}{\sin^2(x-1)}\,dx$

14. $\int_0^9 e^{\ln\sqrt{x}}\,dx$

15. $\int_0^{\frac{\sqrt{2}}{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

16. $\int_0^{\frac{\pi}{12}} \tan 3x \sec^2 3x\,dx$

17. $\int_1^4 \frac{2^{\sqrt{x}}}{2\sqrt{x}}\,dx$

18. $\int_0^{\sqrt{\ln x}} xe^{x^2}\,dx$

19. $\int_0^{\frac{3}{4}} \frac{x}{\sqrt{1-x}}\,dx$

20. $\int_0^1 \frac{x}{\sqrt{8x^2+1}}\,dx$

Sample Problems

Multiple Choice – No Calculator

1. $\dfrac{d}{dx}\int_2^x \ln t\, dt =$

(A) $\ln x$ (B) $\ln 2$ (C) $\dfrac{1}{x}$

(D) $\dfrac{1}{2}$ (E) $\ln x - \ln 2$

2. If $g(x) = \int_\pi^{\pi x} \cos(t^2)\, dt$, then $g'(x) =$

(A) $\sin\left(\pi^2 x^2\right)$ (B) $\pi x \sin\left(\pi^2 x^2\right)$ (C) $\pi x \cos\left(\pi^2 x^2\right)$

(D) $\cos\left(\pi^2 x^2\right)$ (E) $\pi \cos\left(\pi^2 x^2\right)$

3. $\dfrac{d}{dx}\int_{\sin x}^4 \sqrt{1+t^2}\, dt =$

(A) $\sqrt{1+\sin^2 x}$ (B) $-\cos x\sqrt{1+\sin^2 x}$ (C) $-\sqrt{1+\sin^2 x}$

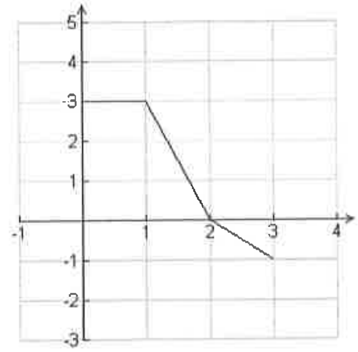
(D) $\cos x\sqrt{1+\sin^2 x}$ (E) $\sqrt{1+\cos^2 x}$

4. If f has two continuous derivatives on [5, 10], then $\int_5^{10} f''(t)\, dt =$

(A) $f'''(10) - f'''(5)$ (B) $f(10) - f(5)$ (C) $f'(10) - f'(5)$

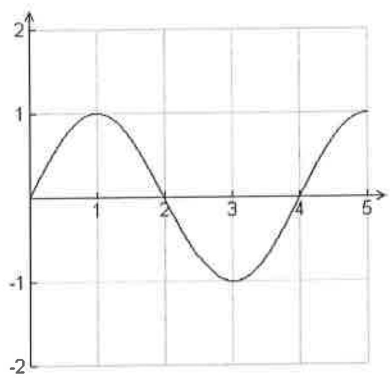
(D) $f''(10) - f''(5)$ (E) $f''(5) - f''(10)$

5. The graph of f is given, and g is an antiderivative of f. If $g(3) = 6$, find $g(0)$.



(A) 1 (B) 2 (C) 4 (D) 5 (E) 10

6. The graph of f is given. $F(x) = \int_0^x f(t)\,dt$



Which of the following statements is true?
(A) F decreases on $(1, 2)$.
(B) F has a relative minimum at $x = 2$
(C) F decreases on $(2, 4)$
(D) F has a relative maximum at $x = 1$.
(E) F has a point of inflection at $x = 4$.

7. $\dfrac{d}{dx}\displaystyle\int_{x}^{x^2} \tan(t)\, dt =$

(A) $\tan\left(x^2\right) - \tan x$

(B) $\tan x - \tan\left(x^2\right)$

(C) $\tan x - 2x\tan\left(x^2\right)$

(D) $2x\tan\left(x^2\right) - \tan x$

(E) $\sec^2\left(x^2\right) - \sec^2 x$

8. $\displaystyle\int_{1}^{e} \left(x - \frac{5}{x}\right) dx =$

(A) $\frac{1}{2}e^2 - \frac{11}{2}$

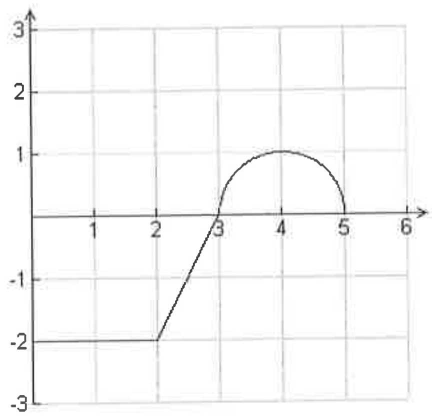
(B) $\frac{1}{2}e^2 - \frac{9}{2}$

(C) $e^2 - \frac{11}{2}$

(D) $\frac{1}{2}e^2 - \frac{3}{2}$

(E) $\frac{11}{2} - \frac{1}{2}e^2$

Free Response 1 – No Calculator



The graph of f is given. It consists of two line segments and a semi-circle.

$g(x) = \int_1^x f(t)\,dt$

(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.

Multiple Choice – Calculator Allowed

1. If $g(x) = \int_0^x \sin^2 t \, dt$, then $g'(2) =$

   (A) 0 (B) 0.001 (C) 0 .173 (D) 0 .827 (E) 1.189

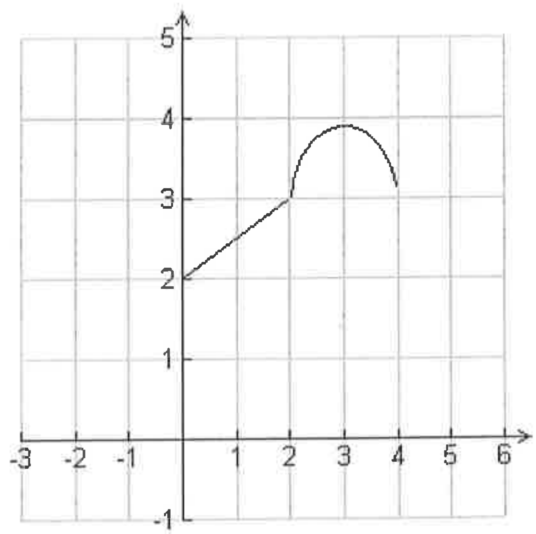
2. A car sold for \$16,000 and depreciated at a rate of $2e^{t^2}$ dollars per year. What is the value of the car 3 years after the purchase?

   (A) \$206.17 (B) \$2889.09 (C) \$13,110.91

   (D) \$16,206.17 (E) \$18,889.09

3. The graph of f is given, and $F(x)$ is an antiderivative of f. If $\int_2^4 f(x)\,dx = 7.5$, find $F(4) - F(0)$.

   (A) 1 (B) 1.5 (C) 7.5 (D) 12.5 (E) 18.5



4. The acceleration of an object in motion is defined by $\sqrt{1+t^2}$. The velocity at $t = 6$ is 22. Find the velocity at $t = 1$.

(A) 1.414 (B) 3.654 (C) 18.346 (D) 22.023 (E) 30.346

5. $h(x) = \int_1^x g(t)\,dt$ and $g(t) = \int_0^{t^2} \frac{\sqrt{1+u^2}}{u}\,du$. Find $h''(2.5)$.

(A) 1.013 (B) 1.077 (C) 2.154 (D) 5.064 (E) 12.659

6. Find $\int_{-2}^{2} f(x)\,dx$ if $f(x) = \begin{cases} 2x^2, & -2 \le x \le 0 \\ \sin 2x, & 0 < x \le 2 \end{cases}$

(A) 0 (B) 4.507 (C) 5.403 (D) 6.161 (E) 10.667

7. Let $g(x)$ be an antiderivative of $\frac{x^3}{\ln x}$. If $g(2) = 3$, find $g(6)$.

(A) 120.552 (B) 123.552 (C) 208.122

(D) 211.122 (E) 214.122

8. $h(x) = \int_0^{2x} (e^{\cos t} - 1)\,dt$ on (3, 6). On which interval(s) is h decreasing?

(A) (3.927, 5.498) (B) (5.498, 6)

(C) (3, 4.712) (D) Always decreasing on (3, 6)

(E) Never decreasing on (3, 6)

Free Response – Calculator Active

Let $g(x) = \int_1^x (5 - 8\sqrt{\ln t})\,dt$ for $x > 1$. Let $h(x) = \int_1^{x^2} (5 - 8\sqrt{\ln t})\,dt$ for $x > 1$.

(a) Write an equation of the tangent to g at $x = 3$.

(b) What is $h'(x)$?

(c) On which open interval(s) is g decreasing? Justify your answer?

(d) Find all x values for which h has relative extrema. Label them as maximum or minimum and justify your answer.

Key to practice problems

1. D
2. E
3. D
4. C
5. C
6. C
7. $\ln|x-1|+C$
8. $\ln|1+e^x|+C$
9. $\tan^{-1}(e^x)+C$
10. $\frac{1}{3}x^3-x-\frac{1}{4x}+C$
11. $\frac{2}{5}\left(\sqrt{x-3}\right)^5+\frac{10}{3}\left(\sqrt{x-3}\right)^3+C$
12. $-\frac{1}{3}(4-2x)^{\frac{3}{2}}+C$
13. $-\csc(x-1)+C$
14. 18
15. $\frac{\pi^2}{32}$
16. $\frac{1}{6}$
17. $\frac{2}{\ln 2}$
18. $\frac{1}{2}(x-1)$
19. $\frac{5}{12}$
20. $\frac{1}{4}$

Key

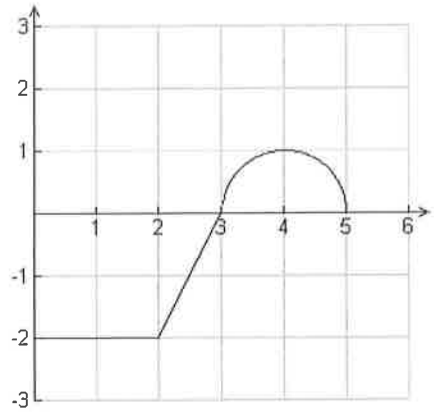
No Calculator

1. A
2. E
3. B
4. C
5. B
6. C
7. D
8. A

Calculator Allowed

1. D
2. C
3. D
4. B
5. D
6. D
7. E
8. A

Free Response 1 – No Calculator



The graph of f is given. It consists of two line segments and a semi-circle.

$g(x)=\int_1^x f(t)\,dt$

(a) Find $g(0)$, $g(1)$, and $g(5)$.

(b) Find $g'(2)$, $g''(2)$, and $g'''(4)$ or state that it does not exist.

(c) For what value(s) of x does the graph of g have a point of inflection? Justify your answer.

(d) Find the absolute maximum and absolute minimum values of g on [0, 5]. Justify your answer.

(a) $g(0)=\int_1^0 f(t)\,dt=2$

$g(1)=\int_1^1 f(t)\,dt=0$

$g(5)=\int_1^5 f(t)\,dt=\frac{1}{2}\pi-3$

(b) $g'(2)=f(2)=-2$

$g''(2)=f'(2)=\text{DNE}$

$g''(4)=f'(4)=0$

(c) g has a point of inflection at $x=4$ because $g'=f$ changes from increasing to decreasing.

(d) Candidates are $x=0, 3, 5$, the endpoints of the interval and the critical number.

x	$g(x)$
0	2
3	-3
5	$\frac{1}{2}\pi-3$

The absolute minimum value is -3.
The absolute maximum value is 2.

2 pts: 1 pt $g(0)$

1 pt $g(1)$ and $g(5)$

2 pts: 1 pt $g''(2)$

1 pt $g'(2)$ and $g''(4)$

2 pts: 1 pt $x=4$

1 pt justification

3 pts: 1 pt for candidates

1 pt evaluating candidates

1 pt for answers

Free Response 1 – Calculator Allowed

Let $g(x) = \int_1^x (5 - 8\sqrt{\ln t})\, dt$ for $x > 1$. Let $h(x) = \int_1^{x^2} (5 - 8\sqrt{\ln t})\, dt$ for $x > 1$.

(a) Write an equation of the tangent to g at $x = 3$.
(b) What is $h'(x)$?
(c) On which open interval(s) is g decreasing? Justify your answer?
(d) Find all x values for which h has relative extrema. Label them as maximum or minimum and justify your answer.

(a) $y + 2.354 = -3.385(x - 3)$	3 pts: 1 pt $g(3) = -2.354$ 1 pt $g'(3) = -3.385$ 1 pt equation
(b) $h'(x) = 2x\left(5 - 8\sqrt{\ln x^2}\right)$	2 pts for $h'(x)$
(c) g is decreasing where $g'(x) < 0$ $g'(x) = 5 - 8\sqrt{\ln t}$ $(1.4779, \infty)$	2 pts: 1 pt correct interval 1 pt justification
(d) h has a relative maximum at $x = 1.2156$ because h' changes sign from positive to negative.	2 pts: 1 pt correct relative maximum 1 pt justification