WU

Parametric MC:

1) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3 + 4t - 8}{3t^2 - 2t}$

C

$3t^2 - 2t = 0$

$t(3t - 2) = 0$

$t = 0, \frac{2}{3}$

2) $s(t) = \int_0^4 \sqrt{(2t)^2 + 1^2}\, dt$ $s'(t) = v(t) = \langle 2t, 1 \rangle$

D

3) $x = e^{4t}$ $y = \sin 6t$

A $\frac{dy}{dx} = \frac{6\cos 6t}{4e^{4t}} = \frac{3e^{-4t}\cos 6t}{2}$

4) $x = 2\sin t$ $y = 3\cos t$

$x' = 2\cos t$ $y' = -3\sin t$

$\int_0^{\frac{\pi}{2}} \sqrt{(2\cos t)^2 + (-3\sin t)^2}\, dt$

$\sqrt{4\cos^2 t + 9\sin^2 t}$

A

$4(1 - \sin^2 t) + 9\sin^2 t$

$4 - 4\sin^2 t + 9\sin^2 t$

$4 + 5\sin^2 t$

5) $x = \frac{4}{3}t^3 - t^2$

B $y = t^5 + t^2 - 7t$

$\frac{5t^4 + 2t - 7}{4t^2 - 2t}$

$4t^2 - 2t = 0$

$2t(2t - 1) = 0$

$t = 0, \frac{1}{2}$

10.4 Polar Coordinates + Graphs

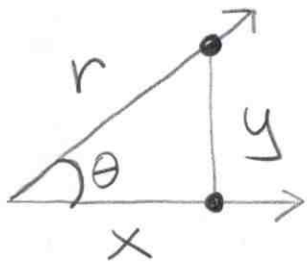
* point on polar graph = (r, θ)

Polar and Rectangular:

$x^2 + y^2 = r^2$

$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$



$R \to P$: need (r, θ)

1) $r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

2) $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$P \to R$: need (x, y)

$\cos\theta = \frac{x}{r}$

1) $x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

2) $y = r\sin\theta$

$R \to P$

1) $x^2 + y^2 - 2x = 0$

$r^2 - 2x = 0$

$r^2 = 2x$

$\frac{r^2}{r} = \frac{2r\cos\theta}{r}$

$r = 2\cos\theta$

2) $xy = 4$

$(r\cos\theta)(r\sin\theta) = 4$

$r^2 = \frac{4}{\cos\theta\sin\theta}$

$r^2 = 4\sec\theta\csc\theta$

$r = \sqrt{4\sec\theta\csc\theta}$

3) $(x^2+y^2)^2 - 9(x^2-y^2) = 0$

$r^4 - 9(r^2\cos^2\theta - r^2\sin^2\theta) = 0$

$r^4 - 9r^2(\cos^2\theta - \sin^2\theta) = 0$

$r^2[r^2 - 9(\cos^2\theta - \sin^2\theta)] = 0$

$r^2 - 9\cos^2\theta + 9\sin^2\theta = 0$

$r^2 = 9\cos^2\theta - 9\sin^2\theta$

4) $(-1, 1)$ → Quad II

$r = \sqrt{x^2+y^2}$

$r = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) = \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

P → R

1) $r = 3\sec\theta$

$r = \dfrac{3}{\cos\theta}$

$3 = r\cos\theta$

$3 = x$

2) $r = -\sec\theta\tan\theta$

$r = \dfrac{-1}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$

$r = -\dfrac{\sin\theta}{\cos^2\theta}$

$r = \dfrac{-\frac{y}{r}}{\frac{x^2}{r^2}}$

$r = \dfrac{-y}{r} \cdot \dfrac{r^2}{x^2}$

$r = -\dfrac{yr}{x^2}$

$1 = \dfrac{-y}{x^2}$

$y = -x^2$

3) $r = 5\cot\theta\csc\theta$

$r = 5\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\sin\theta}$

$r = \dfrac{5\cos\theta}{\sin^2\theta}$

$r = \dfrac{\frac{5x}{r}}{\frac{y^2}{r^2}}$

$r = \dfrac{5x}{r} \cdot \dfrac{r^2}{y^2}$

$r = \dfrac{5xr}{y^2}$

$1 = \dfrac{5x}{y^2}$

$y^2 = 5x$

$\frac{1}{5}y^2 = x$

4) $(\sqrt{3}, \frac{\pi}{6})$

$x = \sqrt{3}\cos\frac{\pi}{6} = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2}$

$y = \sqrt{3}\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Graphs : Pg 735 (handout)

* symmetry:
cos x-axis
sin y-axis

Slope in Polar Form:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

1) Find slope.

$r = 3\cos\theta + 6\sin\theta \qquad \theta = 0$

$$\frac{dy}{dx} = \frac{(3\cos\theta + 6\sin\theta)(\cos\theta) + (-3\sin\theta + 6\cos\theta)(\sin\theta)}{-(3\cos\theta + 6\sin\theta)(\sin\theta) + (-3\sin\theta + 6\cos\theta)(\cos\theta)}$$

$$\frac{dy}{dx} = \frac{(3)(1) + (6)(0)}{0 + 6} = \frac{3}{6} = \frac{1}{2}$$

2) Eqn of tangent line at $(2, 0)$
$\qquad\qquad\qquad\qquad\quad r \ \ \theta$

$r = 2(1 - \sin\theta)$

$r' = 2(-\cos\theta)$

$f(0) = 2$

$f'(0) = -2$

$\sin 0 = 0$

$\cos 0 = 1$

$$\frac{dy}{dx} = \frac{2(1) + -2(0)}{-2(0) + -2(1)} = \frac{2}{-2} = -1 \quad \text{(m)}$$

$y - 0 = -1(x - 2)$

$y = -x + 2$

$x = 2\cos 0 = 2$

$y = 2\sin 0 = 0$

pt : $(2, 0)$

~~3)~~ Pts of horizontal/vertical tangents of $r = a\sin\theta$

$$\frac{dy}{dx} = \frac{a\sin\theta\cos\theta + a\cos\theta\sin\theta}{-a\sin\theta\sin\theta + a\cos\theta\cos\theta}$$

HT:

$$a\sin\theta\cos\theta + a\sin\theta\cos\theta = 0$$

$$\cancel{2a}\sin\theta\cos\theta = 0$$

$$\sin\theta\cos\theta = 0$$

$$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$\theta = 0, r = 0$

$\theta = \frac{\pi}{2}, r = a$

$\theta = \pi, r = 0$

$\theta = \frac{3\pi}{2}, r = -a$

$\left(a, \frac{\pi}{2}\right)$

$\left(-a, \frac{3\pi}{2}\right)$

VT:

$$-a\sin^2\theta + a\cos^2\theta = 0$$

$$\sin^2\theta = \cos^2\theta$$

$$\theta = \frac{\pi}{4}, \ldots$$

$$r = \frac{a\sqrt{2}}{2}$$

$$\left(\frac{a\sqrt{2}}{2}, \frac{\pi}{4}\right)$$

4) $r = 2(1 - \sin\theta)$

YT Find slope @ $(4, \frac{3\pi}{2})$ (r, θ)

$r = 2 - 2\sin\theta$ $\qquad f(\frac{3\pi}{2}) = 4$

$r' = -2\cos\theta$ $\qquad f'(\frac{3\pi}{2}) = 0$

$\sin\frac{3\pi}{2} = -1$

$\cos\frac{3\pi}{2} = 0$

$$\frac{dy}{dx} = \frac{4(0) + 0(-1)}{-4(-1) + 0(0)} = \frac{0}{4} = 0$$

5) Find pts where vertical tangents are to graph: $r = 2(1 - \sin\theta)$

$2 - 2\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \rightarrow = 0$$

$$\frac{dx}{d\theta} = (-2 + 2\sin\theta)\sin\theta + -2\cos\theta\cos\theta$$

$$= -2\sin\theta + 2\sin^2\theta - 2\cos^2\theta = 0$$

$$-2\sin\theta + 2\sin^2\theta - 2(1 - \sin^2\theta) = 0$$

$$-2\sin\theta + 2\sin^2\theta - 2 + 2\sin^2\theta = 0$$

$$4\sin^2\theta - 2\sin\theta - 2 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$\sin\theta = -\frac{1}{2}$ $\qquad \sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\qquad \theta = \frac{\pi}{2} \rightarrow \frac{dy}{d\theta} = 0$ X

$(3, 7\pi/6)$

$(3, 11\pi/6)$

Arc Length of Polar Curve:

$$S = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

1) $[0, 2\pi]$ $\quad r = 2 - 2\cos\theta$

$$\frac{dr}{d\theta} = 2\sin\theta$$

$$\int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta = \boxed{16}$$

2) Golden Spiral: 2 revolutions

$$r = 1.618013\, e^{0.30635\theta}$$

$$r^2 = 2.61797\, e^{0.6127\theta}$$

$$\frac{dr}{d\theta} = .49568\, e^{0.30635\theta}$$

$$\int_0^{4\pi} \sqrt{2.61797 e^{0.6127\theta} + .2457 e^{0.6127\theta}}\, d\theta$$

$$= 254.0$$