AP Calculus BC Intro to Polar Name: Key

Polar Coordinates: (r, θ) Rectangular Coordinates (x, y)

Converting Polar ←→ Rectangular: Use the following formulas.

- $x^2 + y^2 = r^2$ or $r = \sqrt{x^2 + y^2}$
- $\cos\theta = \frac{x}{r}$ or $x = r\cos\theta$
- $\sin\theta = \frac{y}{r}$ or $y = r\sin\theta$
- $\tan\theta = \frac{y}{x}$ or $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

You Try: Convert the Rectangular Coordinates or Equations into Polar Coordinates or Equations.

1. $x^2 + y^2 - 2x = 0$

$r^2 - 2x = 0$
$r^2 - 2r\cos\theta = 0$
$r^2 = 2r\cos\theta$
$r = 2\cos\theta$

2. $xy = 4$

$(r\cos\theta)(r\sin\theta) = 4$
$r^2 = \frac{4}{\cos\theta\sin\theta}$
$r^2 = 4\sec\theta\csc\theta$
$r = \pm 2\sqrt{\sec\theta\csc\theta}$

3. $(x^2 + y^2)^2 - 9(x^2 - y^2) = 0$

$r^4 - 9(r^2\cos^2\theta - r^2\sin^2\theta) = 0$
$r^4 - 9r^2\cos^2\theta + 9r^2\sin^2\theta = 0$
$r^2(r^2 - 9\cos^2\theta + 9\sin^2\theta) = 0$
$r^2 = 9\cos^2\theta - 9\sin^2\theta$
$r = \pm\sqrt{9\cos^2\theta - 9\sin^2\theta}$

4. (-1, 1) Quad II

$r = \sqrt{x^2 + y^2}$
$r = \sqrt{2}$
$\theta = \tan^{-1}(-1) = \frac{3\pi}{4}$
$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

You Try: Convert the Polar Coordinates or Equations into Rectangular Coordinates or Equations.

1. $r = 3\sec\theta$

$r = \frac{3}{\cos\theta}$
$3 = r\cos\theta$
$x = 3$

2. $r = -\sec\theta\tan\theta$

$r = -\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{-\sin\theta}{\cos^2\theta}$
$r = \frac{-\frac{y}{r}}{\frac{x^2}{r^2}} = \frac{-y}{r} \cdot \frac{r^2}{x^2}$
$r = \frac{-yr}{x^2}$
$rx^2 = -yr$
$rx^2 + yr = 0$
$r(x^2 + y) = 0$
$y = -x^2$

$r\cos\theta = -\tan\theta$
$x = -\frac{y}{x}$ $y = -x^2$

3. $r = 5\cot\theta\csc\theta$

$r = 5\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} = \frac{5\cos\theta}{\sin^2\theta}$
$r = \frac{\frac{5x}{r}}{\frac{y^2}{r^2}} = \frac{5x}{r} \cdot \frac{r^2}{y^2}$
$r = \frac{5xr}{y^2}$
$ry^2 = 5xr$
$y^2 = 5x$

$r\sin\theta = 5\frac{\cos\theta}{\sin\theta}$
$y = 5 \cdot \frac{x}{\cancel{r}} \cdot \frac{\cancel{r}}{y}$
$y = \frac{5x}{y}$
$y^2 = 5x$

4. $\left(\sqrt{3}, \frac{\pi}{6}\right)$

$x = \sqrt{3}\cos\frac{\pi}{6} = \frac{3}{2}$
$y = \sqrt{3}\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

$r=f(\theta)$

$y=r\sin\theta$
$x=r\cos\theta$

Slope in Polar Form: $\dfrac{dy}{dx}=\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\dfrac{f(\theta)\cos\theta+f'(\theta)\sin\theta}{-f(\theta)\sin\theta+f'(\theta)\cos\theta}$

$$\frac{r\cos\theta+\sin\cdot r'}{-r\sin\theta+\cos\cdot r'}$$

You Try: *check slope in calc

1. Find the slope of $r=3\cos\theta+6\sin\theta$ at $\theta=0$. $r'=-3\sin\theta+6\cos\theta$ $r(0)=3$ $r'(0)=6$ $\sin 0=0$ $\cos 0=1$ $\frac{dy}{dx}=\frac{3(1)+0(6)}{-3(0)+1(6)}$ $\frac{3}{6}=\frac{1}{2}$	2. Find the equation of the tangent line to $r=2(1-\sin\theta)$ at (2, 0). $r=2-2\sin\theta$ $(2,0)$ $r\ \theta$ $r'=-2\cos\theta$ $r(0)=2$ $r'(0)=-2$ $\sin 0=0$ $\cos 0=1$ $\frac{dy}{dx}=\frac{2(1)+0(-2)}{-2(0)+1(-2)}=\frac{2}{-2}=-1$ $x=r\cos\theta$ $y=r\sin\theta$ $x=2\cos 0$ $y=2\sin 0$ $x=2$ $y=0$ $y=-1(x-2)$
3. Find the slope of $r=2(1-\sin\theta)$ at $\left(4,\frac{3\pi}{2}\right)$. $2-2\sin\theta$ $r'=-2\cos\theta$ $r(3\pi/2)=4$ $r'(3\pi/2)=0$ $\sin(3\pi/2)=-1$ $\cos(3\pi/2)=0$ $\frac{dy}{dx}=\frac{4(0)+-1(0)}{-4(-1)+0(0)}$ $=\frac{0}{4}=0$ $x=4\cos\frac{3\pi}{2}$ $y=4\sin\frac{3\pi}{2}$ $x=0$ $y=-4$	4. Find the points where there are vertical tangents to the graph of: $r=2(1-\sin\theta)$. $r'=-2\cos\theta$ $\frac{dx}{d\theta}=0$ $-(2-2\sin\theta)(\sin\theta)+\cos\theta(-2\cos\theta)=0$ $(-2+2\sin\theta)(\sin\theta)-2\cos^2\theta=0$ $-2\sin\theta+2\sin^2\theta-2\cos^2\theta=0$ $-2\sin\theta+2\sin^2\theta-2(1-\sin^2\theta)=0$ $-2\sin\theta+2\sin^2\theta-2+2\sin^2\theta=0$ $4\sin^2\theta-2\sin\theta-2=0$

$2(2\sin^2\theta-\sin\theta-1)=0$
$2(2\sin\theta+1)(\sin\theta-1)=0$
$2\sin\theta+1=0$ $\sin\theta=1$
$\sin\theta=-\frac{1}{2}$ $\cancel{\theta=\frac{\pi}{2}}$
$\theta=\frac{7\pi}{6},\ \frac{11\pi}{6}$ not diff. there!

Arc Length of a Polar Curve:

$$s=\int_{\alpha}^{\beta}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

You Try: *not AP

1. Find the length of the arc from $[0,2\pi]$ for $r=2-2\cos\theta$. $r'=2\sin\theta$ $\int_0^{2\pi}\sqrt{(2-2\cos\theta)^2+(2\sin\theta)^2}\,d\theta$ $=16$ $\frac{dy}{d\theta}=0$ check: $(2-2\sin\theta)(\cos\theta)+(\sin\theta)(-2\cos\theta)$